Coordinated Freight Routing with Participation Incentives for Multiple Classes of Users

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Outline

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Motivation
Motivation (1)

- In the **absence of cooperation**, the users of the transportation network act independently in an effort to minimize their own individual cost (e.g. travel time).

- This situation is known as **User Equilibrium (UE)** where no driver has an incentive to unilaterally change his/her route selection since he/she is not going to have a benefit from such a change.

- The situation where the network users **cooperate** in a manner which contributes to the minimization of a “social cost” function (e.g. total travel time) is known as **System Optimum (SO)**.

- The inefficiency of the UE compared to the SO in terms of the total social cost has been addressed to the literature as the **Price of Anarchy (PoA)**.
Motivation (2)

- In **realistic transportation networks**, the **PoA** has been shown to reach the **value of 2 or even greater**, indicating the necessity for its reduction.

- In a **SO solution**, some drivers may get harmed while some others may get benefit compared to the **UE** leading to **unfair** situations.

- In **realistic transportation scenarios**, every driver has his/her own **Value-of-Time (VOT)**

- Due to **lack of cooperation**, truck drivers are not aware of the real-time demand for trips of the rest truck drivers.
• **Question:** Does there exist a way to route the drivers in a manner where they do not get harmed compared to the UE leading to a fair situation while concurrently making the transportation network to approach as close as possible the SO solution?

• **Answer:** Use Game Theory and appropriate pricing schemes
Previous Work
Previous Work

• **Congestion Pricing**: Assign fees to each route to incentivize drivers to follow routes so that the network is driven to the SO solution

• **Uniform Revenue Refunding**: Share the money collected from congestion pricing uniformly among the drivers

• **Question**: What happens in the case of a non-deterministic scenario?

• **Question**: Can we make something more fair by pricing the truck drivers according to their Value of Time (VOT)?
Problem Formulation
Game Theoretic Formulation

- Use of a **non-atomic, symmetric information** game theoretic model assuming a **stochastic demand vector** for the truck drivers while the coordinator knows the exact realization of the demand.

- The **truck drivers** are considered to be the “**players**” of the game and their objective is to **minimize** their **own individual travel time**.

- **Coordinator** receives the Origin-Destination demands and additionally asks the truck drivers to declare their Value of Time (VOT), and then provides routing instructions by **minimizing** a “**social cost**” function.
Mechanism Design Criteria

• Truck drivers should have a lower individual travel time compared to the UE in order to provide them individual incentives for participation.

• Since the mechanism will compensate truck drivers who get harmed compared to the UE according to their declared VOT, guaranteeing that they will truthfully declare their VOT is of high importance in order to avoid the exploitability of the mechanism.

• The resulting monetary scheme should be budget balanced on average.

• Drive the network as close as possible to the SO solution.
Mathematical Models
User Equilibrium (UE)

\[
\begin{align*}
\text{minimize} \quad & \lambda E[T_{tr}(\alpha)] + (1 - \lambda) E[T_{tr}^{mon}(\alpha)] \\
\text{subject to} \quad & 0 \leq \alpha^j_{w,r} \perp F^w_{j,r}(\alpha) - \delta^j_w \geq 0, \; \forall j, w, r \\
& \sum_{r \in R_j} \alpha^j_{w,r} = 1, \; \forall j, w
\end{align*}
\]

- Minimize a weighted combination of total travel time and total monetary cost of trucks

- Due to lack of cooperation, the truck drivers do not know the actual realization of the demand for the rest truck drivers
System Optimum (SO)

\[
\begin{align*}
\text{minimize} \quad & \lambda E[T_s(\alpha)] + (1 - \lambda) E[T_{tr}^{mon}(\alpha)] \\
\text{subject to} \quad & \sum_{r \in R_j} \alpha_{w,r}^{c,j} = 1, \forall c, j, w \\
& \alpha_{w,r}^{c,j} \geq 0, \forall c, j, w, r
\end{align*}
\]

- Minimize a weighted combination of the total travel time of the network and the total monetary cost of trucks
Minimize a weighted combination of the total travel time of the network and the total monetary cost of trucks.
VOT Based Pricing

\[
\begin{align*}
\text{minimize} & \quad \lambda E[T_s(\alpha)] + (1 - \lambda) E[T_{tr}^{mon}(\alpha)] \\
\text{subject to} & \quad \sum_c \sum_{r \in R_j} p_c \alpha_{w,r}^{c,j} (J_{w,r}^{M,c,j} + \frac{1}{s_w} \tau_{w,r}^{c,j}) \leq \sum_c p_c A_{c,j}^{UE}, \quad \forall j, w \\
& \quad \sum_c \sum_{r \in R_j} p_c \alpha_{i,r}^{c,j} (J_{i,r}^{M,c,j} + \frac{1}{s_i} \tau_{i,r}^{c,j}) \leq \sum_c \sum_{r \in R_j} p_c \alpha_{k,r}^{c,j} (J_{k,r}^{M,c,j} + \frac{1}{s_i} \tau_{k,r}^{c,j}), \quad \forall j, i, k \\
& \quad \sum_{c} \sum_{j=1}^{N} \sum_{w=1}^{N} \sum_{r \in R_j} p_c d_{c,j}^{rw} \alpha_{w,r}^{c,j} \tau_{w,r}^{c,j} = 0 \\
& \quad \sum_{r \in R_j} \alpha_{w,r}^{c,j} = 1, \quad \forall c, j, w \\
& \quad \alpha_{w,r}^{c,j} \geq 0, \quad \forall c, j, w, r
\end{align*}
\]
Simulation Results
Why not Congestion Pricing? (1)

Scenario 1:
\[ d = 28 \] (deterministic)

Scenario 2:
\[ d = 28 w.p. 0.5 \]
\[ d = 37 w.p. 0.5 \]

\[ C_{1T} = 20 + X_{1T}^2 \]
\[ C_{2T} = 2 + 3X_{2T}^2 \]

\[ s_1 = 100 \text{ $/hr} \]
\[ s_2 = 30 \text{ $/hr} \]
Why not Congestion Pricing? (2)

\[ C_{1T} = 20 + X_{1T}^2 \]
\[ C_{2T} = 2 + 3X_{2T}^2 \]

\[ d(s_1) = \begin{cases} 2, & w.p. 0.5 \\ 5, & w.p. 0.5 \end{cases} \]
\[ d(s_2) = \begin{cases} 4, & w.p. 0.5 \\ 8, & w.p. 0.5 \end{cases} \]

Scenario 1: \( s_1 = 30 \text{ $/hr} \), \( s_2 = 30 \text{ $/hr} \)
Scenario 2: \( s_1 = 50 \text{ $/hr} \), \( s_2 = 30 \text{ $/hr} \)
Scenario 3: \( s_1 = 100 \text{ $/hr} \), \( s_2 = 30 \text{ $/hr} \)
Scenario 4: \( s_1 = 200 \text{ $/hr} \), \( s_2 = 30 \text{ $/hr} \)
• Application to the benchmark Sioux Falls network (24 nodes, 76 links)

• We assume that the truck drivers have only 6 available OD pairs: (1,7), (1,11), (10,11), (10,20), (15,5), (24,10)

• We have 2 classes of users with the following VOT:

  \[ s_1 = 200 \text{ } \$/hr \]

  \[ s_2 = 50 \text{ } \$/hr \]
• The number of passenger vehicles at each link is considered to be constant

• The cost of each road segment is given by a BPR function (travel time):

\[ C_{lp}^t(X_{lp}^t, X_{lT}^t) = C_{lT}^t(X_{lp}^t, X_{lT}^t) = \delta_a + \delta_b \left( \frac{X_{lp}^t + 3X_{lT}^t}{\delta_k} \right) \]

• The demand takes one of the following two equiprobable values:

\[ d_1 = \begin{bmatrix} 3 & 4.5 & 6 & 3 & 14 & 3.6 \\ 1 & 2.8 & 5.4 & 7 & 9 & 2 \end{bmatrix} \]

\[ d_2 = \begin{bmatrix} 5 & 1.8 & 3.9 & 15 & 6.4 & 2.4 \\ 6 & 5.5 & 1.8 & 6.5 & 11 & 6 \end{bmatrix} \]
Simulation Results

<table>
<thead>
<tr>
<th></th>
<th>UE</th>
<th>SO</th>
<th>CPURR</th>
<th>VOT-BP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[T_{tr}]$</td>
<td>53,574.4</td>
<td>49,082.5</td>
<td>53,570.3</td>
<td>49,049.8</td>
</tr>
<tr>
<td>$E[T_{tr}^{mon}]$</td>
<td>117,941.4</td>
<td>104,445.5</td>
<td>117,932.6</td>
<td>104,647.5</td>
</tr>
<tr>
<td>$E[T_s]$</td>
<td>167,160.7</td>
<td>157,924.8</td>
<td>167,152.0</td>
<td>157,934.9</td>
</tr>
</tbody>
</table>

- **VOT-BP** achieves 8.45% reduction in $E[T_{tr}]$, 11.27% reduction in $E[T_{tr}^{mon}]$ and 5.52% reduction in $E[T_s]$ while approaching the SO solution

- **CPURR** cannot “escape” the UE solution
Conclusion

• Congestion Pricing with Uniform Revenue Refunding (CPURR) cannot “escape” the UE solution in the absence of cooperation between the truck drivers

• CPURR is inefficient when different drivers have different VOT

• We can ask the truck drivers to declare their VOT and guarantee incentive compatibility

• Value-of-Time based Pricing can approach the SO solution while concurrently providing individual participation incentives
End of Presentation

Questions ?