SOLVING LARGE-SCALE TRAFFIC ASSIGNMENT PROBLEM
WITH A DISTRIBUTED, SIMULATION-BASED LOAD BALANCING METHOD

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Global Transhipment

Jean-Paul Rodrigue (2017), Maritime Transportation, New York: Routledge
Los Angeles Metropolitan Area

US Federal Gov, Map of Los Angeles, California
Research Problem

To assign a number of demands from origins to destinations with considerations of several factors: time window, road network traffic condition, emissions, etc.

- **Time window**: pick up time window, delivery time window
- **Road network traffic condition**: Vehicle Travel Time
- **Emissions**: $CO_2$, $PM_{2.5}$, $NO_x$
Formulation 1

\[
\min_x \sum_a \int_0^{v_a} S_a(x)dx
\]

subject to:

\[
v_a = \sum_i \sum_j \sum_r \alpha_{ij}^arx_{ij}^r
\]

\[
\sum_r x_{ij}^r = T_{ij}
\]

\[
v_a \geq 0, x_{ij}^r \geq 0
\]

- \(x_{ij}^r\) is the number of vehicles on path \(r\) from origin \(i\) to destination \(j\)
- \(\alpha_{ij}^ar = 1\) if link \(a\) is on path \(r\) from \(i\) to \(j\); zero otherwise
- \(S_a(x)\): volume delay function, stating the relationship between resistance and volume of traffic, usually we use Bureau of Public Roads (BPR) model:

\[
S_a(v_a) = t_a(1 + 0.15 \left( \frac{v_a}{c_a} \right)^4)
\]

- \(t_a\): free flow travel time on link \(a\) per unit of time
- \(v_a\): volume of traffic on link \(a\) per unit of time
- \(c_a\): capacity of link \(a\) per unit of time

The solution to this nonlinear programming problem reaches such condition that travelers will strive to find the shortest path from origin to destination and network equilibrium occurs when no traveler can decrease travel delay/resistance by shifting to a new path.
Formulation 2

\[
\begin{align*}
\min \text{TC}(X) &= \sum_{k \in K} \sum_{i \in I} \sum_{j \in J} \sum_{r \in R_{i,j}} S_{i,j}^r(k) X_{i,j}^r(k) \\
&= \sum_{k \in K} \sum_{i \in I} \sum_{j \in J} \sum_{r \in R_{i,j}} (C_{i,j}^r(k) + \theta^r T_{i,j}^r(k)) X_{i,j}^r(k)
\end{align*}
\]

subject to:

\[
\begin{align*}
\sum_{k \in K} \sum_{r \in R_{i,j}} X_{i,j}^r(k) &= d_{i,j}, \forall i \in I, \forall j \in J \ (1) \\
\sum_{i \in I} \sum_{j \in J} \sum_{r \in R_{i,j}} \sum_{\tau \leq k} X_{i,j}^r(\tau) \delta_{l,\tau,k} &= x_l(k), \forall l \in L, \forall k \in K \ (2) \\
0 &\leq x_l(k) \leq u_l v_l(k), \forall l \in L^R, \forall k \in K \ (3) \\
X_{i,j}^r(k) &\geq 0, \forall i \in I, \forall j \in J, \forall k \in K \ (4)
\end{align*}
\]

- \(d_{i,j}\): The total demand from an origin \(i\) to a destination \(j\);
- \(X_{i,j}^r(k)\): The freight demand in units of containers from an origin \(i\) to a destination \(j\) using a route \(r\) with a departure time \(k\);
- \(x_l(k)\): The number of containers using edge \(l\) at time \(k\);
- \(u_l\): The edge capacity in units of vehicles for edge \(l\);
- \(v_l(k)\): The vehicle capacity in units of containers per freight vehicle for edge \(l\);
- \(S_{i,j}^r(k)\): combination of the non-travel time vehicle cost \(C_{i,j}^r(k)\) and the cost of the route travel time \(T_{i,j}^r(k)\);
Methodology: general structure

1. Relaxed Problem

\[
\min TC(X) + \sum_{k \in K} \sum_{l} \sigma_l \phi(x_l(k), u_l(k), v_l(k))
\]

subject to constraints (1), (2), (4)

2. Check convergence, if not convergent, proceed to step 3, otherwise, terminate the algorithm and output \(X\) as the optimal solution

\[
\phi(x_l(k), u_l(k), v_l(k)) \leq \eta
\]

3. Increase penalty factor \(\sigma_l\), proceed to step 1
Methodology: solving relaxed problem

- Initial feasible solution $X_{ij}^{(0)}$, transfer it into $x_l$
- Simulate $X$ with road network simulator,
  output simulated results: travel time, emissions, etc.
- Update marginal cost of links and routes in $R_{ij}$ based on simulated results: travel time, emissions, etc.
- Generate one minimum marginal cost route for each $R_{ij}$
- Generate augmented $X_{aug}$ based on $R_{ij}$ in the previous step
- Set new $X$ as $X^{(m+1)} = X^{(m)} + \beta (X_{aug} - X^{(m)})$
- Check convergence, if convergent, output $X$ as the solution of relaxed problem, else, proceed to step 2.
Methodology: service network

- Iterate every edge in the network takes computational time and memory
- 2-layer: service network + road network
- Global optimum to local optimum
- Denote $l$ to service link, $a$ road arc

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Methodology: marginal cost

- Marginal cost: the change in total cost if we add one truck at time instance $k$ on service link $l$

\[
MC_{P_{l'}^p}(k') \approx c_{l'}^p + \sigma_{l'}^p t_{l'}^p(k)
+ \sum_{n_{p'}=1}^{N_{p'}} \left( \sigma_{l'}^p \gamma_{l'}^p \left( e_{a_{p'},n_{p'}}(k') \right) \right) \frac{1}{v_{l'} \left( e_{a_{p'},n_{p'}}(k') \right) \Delta t} \frac{\partial w_{a_{p'},n_{p'}}}{\partial z_{a_{p'},n_{p'}}} \left( e_{a_{p'},n_{p'}}(k') \right)
\]

- $e_{a_{p'},n_{p'}}(k')$: entering time at arc $a_{p'},n_{p'}$, for a freight vehicle using path $p'$ with a departure time of $k'$ from the origin.
- $w_{a_{p'},n_{p'}}(k')$: the travel time of arc $a_{p'},n_{p'}$ at time $k'$
- $z_{a_{p'},n_{p'}}(k')$: the traffic volume on road network arc $a_{p'},n_{p'}$ at time $k'$
Methodology: tackle scalability issue

Marginal cost update depends on simulated results from road network simulator. However, the time and space complexity grows exponentially with:
• the number of demands
• the scale of road network

Solution:
• Distributed network
• Parallel computing
Methodology: with distributed subnetwork

- Initial feasible solution $X^{(0)}$
- Simulate input to subnetwork $X_1, X_2, ...$ with road network simulator $S_1, S_2, ...$, output simulated results: travel time, emissions, etc.
- Update marginal cost of links and routes in $R_{ij}$ based on simulated results: travel time, emissions, etc.
- Generate one minimum marginal cost route for each $R_{ij}$
- Generate augmented $X_{aug}$ based on $R_{ij}$ in the previous step
- Set new $X$ as $X^{(m+1)} = X^{(m)} + \beta (X_{aug} - X^{(m)})$
- Check convergence, if convergent, output $X$ as the solution of relaxed problem, else, proceed to step 2.
Numerical Results

- Baseline Scenario (Long Beach Area)
  - Terminal, Destinations with demand label
  - Routes: Road Network

- Evaluation Scenarios
  - Centralized method
  - Distributed-COSMO

openstreetmap.org
### Numerical Results

<table>
<thead>
<tr>
<th>Method</th>
<th>Total Iterations</th>
<th>Computation Time (Seconds)</th>
<th>Total Vehicle Travel Time (Million Seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centralized Method</td>
<td>3,600</td>
<td>16,240</td>
<td>6.73</td>
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<tr>
<td>Centralized Method</td>
<td>400</td>
<td>464</td>
<td>9.34</td>
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<tr>
<td>2/2 Distributed COSMO</td>
<td>120</td>
<td>160</td>
<td>14.96</td>
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<tr>
<td>5/5 Distributed COSMO</td>
<td>750</td>
<td>600</td>
<td>12.73</td>
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<tr>
<td>10/10 Distributed COSMO</td>
<td>3,000</td>
<td>2,093</td>
<td>11.60</td>
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<td>20/20 Distributed COSMO</td>
<td>12,000</td>
<td>8,169</td>
<td>10.80</td>
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<td>50/50 Distributed COSMO</td>
<td>75,000</td>
<td>50,632</td>
<td>10.06</td>
</tr>
</tbody>
</table>

- **Graphs:**
  - **Total Iterations**
  - **Computation Time (Seconds)**
  - **Total Vehicle Travel Time (Million Seconds)**
Numerical Results

- **Baseline Scenario (the Greater Los Angeles Area)**
  - Terminal, Destinations with demand label
  - Routes: Road Network
  - 5 Sub-level service network

Santa Monica-Downtown LA

Covena

San Bernardino

Long Beach

Irvine
Numerical Results

Data Source

- Southern California Association of Governments (SCAG)
- Warehouse data: Longitudinal Employer-Household Dynamics datasets, 2003 and 2015
- Customer data: Longitudinal Employer-Household Dynamics datasets, 2003 and 2015

Evaluation Scenarios

- Centralized method: too much computational load, cannot accomplish outputting an assignment for freight transportation system
- Distributed-COSMO(Com-Simulation Optimization) with warehouse/customer datasets (2002/2003, 2003/2015, 2015/2003, 2015/2015) and different outbound rates (0, 20%, 40%, 60%, 80%, 100%), to east through San Bernardino
Numerical Results

<table>
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<tr>
<th></th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
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<tbody>
<tr>
<td>2003 warehouse</td>
<td>2.92E+12</td>
<td>4.36E+12</td>
<td>6.21E+12</td>
<td>7.95E+12</td>
<td>9.83E+12</td>
<td>1.16E+13</td>
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<tr>
<td>2003 customer</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>2015 warehouse</td>
<td>3.04E+12</td>
<td>4.51E+12</td>
<td>6.56E+12</td>
<td>8.38E+12</td>
<td>1.02E+13</td>
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<tr>
<td>2015 customer</td>
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<td>2015 customer</td>
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<tr>
<td>2015 warehouse</td>
<td>2.68E+12</td>
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<tr>
<td>2015 customer</td>
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</tbody>
</table>

2015 customer demand: 1667070

2003 customer demand: 1755545

• With increasing outbound demand, decentralized distribution of warehouses tend to have a lower cost on vehicle travel time.
Conclusions

• We solved a large-scale traffic assignment problem with a distributed, simulation-based load balancing method
• The system incorporates road network, service networks and load balancing algorithm
• The method is experimentally presented to provide a local optimal solution for a large-scale traffic assignment problem

Future work

• Subnetworks: optimal partitioning of road network based on multiple factors
• Load balancing strategy: reinforcement learning
• New technologies on freight vehicle industries: electric, hybrid, hydrogen, etc.
• Real-time assignment and much demanding time window
Thank you!