Data Driven Feedback Control of Urban Traffic Systems with Performance Guarantees

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Abstract

We study stability of traffic flow under output feedback ramp metering inspired by joint model predictive control and moving horizon estimation. The running and terminal costs are linear in cell densities, the terminal set is the uncongested region, and the output comprises of density measurements from a subset of the cells. For the cell transmission model over a line network, we provide sufficient conditions on the subset of measurements, the estimation and control horizons, and the inflows at the ramps under which the traffic system is input to state stable. Our analysis relies heavily on the monotonicity property of the traffic flow dynamics.

Contents

1 Disclosure

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2 Acknowledgements

This research was performed in collaboration with Z. Li, and P. Hosseini, who are currently or were formerly associated with the University of Southern California.

3 Introduction

The wide spread use of personal mobile devices and the increasing penetration of connected vehicles are giving rise to new sensing modalities in urban traffic systems, in addition to traditional ones consisting primarily of loop detectors and cameras. There has been a growing interest in estimating traffic states from such measurements. Accurate estimates however require a large number of measurements or a stationary environment. A rigorous understanding of how different combinations of measurement quality, traffic dynamics and control strategy affect performance, e.g., in terms of travel time, is lacking in general. The objective of this proposal is to lay the foundations of a framework that informs *joint* optimal choice of estimation *and* control algorithms, under given sensing modes. The approach can then be further built upon to optimize resource allocation for traffic sensing infrastructure. Specifically, we develop output feedback ramp metering strategies for control of freeways, with provable performance guarantees.

4 Stability Analysis of the Cell Transmission Model under MPC-MHE Ramp Metering

Feedback ramp metering control uses sensing, typically from the freeway mainline, to modulate inflow from the ramps into mainline. There are two main paradigms: performance evaluation of a particular control policy, e.g., ALINEA, or designing a control policy which optimizes a given performance criterion, e.g., see [1]. This project adopts the latter approach, which is also referred to as the *model predictive control (MPC) approach.*

The existing control algorithms, which come with performance guarantees, typically require traffic states of density or queue lengths as inputs. A common approach then is to first estimate the state from sensor measurements and then input it to control algorithms. However, rigorous performance analysis of such a sequential approach is lacking. Estimating traffic state from sensor measurements has attracted sustained research from the transportation science research community, e.g., see [2–4]. While there has been recent interest in designing control algorithms that can directly use sensor measurements, e.g., see [5] and an overview in [6], analytical performance evaluation is lacking.

The paradigm of directly using sensor measurements for control is known as output feedback in control theory. An overview of linear time-invariant systems is provided in [7]. A modern overview of MPC is provided in [8]. Stability issues under state feedback are covered in [9, 10]. Independent state estimation in the context of MPC is discussed in [11]. Output feedback MPC for constrained linear systems is presented in [12]. Computational techniques underlying the moving horizon control and estimation in the context of MPC is provided in [13]. Recently, joint estimation and control with stability guarantees were provided in [14]; however, this result is only initial, and more importantly, their implications for urban traffic systems under modern or traditional sensing modalities are not understood yet. In this study, we exploit the properties of cell transmission models and design an MPC controller with simultaneous state estimation in a similar spirit to [14].

The main contributions of the project are as follows. First, we propose an MPC controller with simultaneous state estimation that only relies on partial information of traffic states. Second, we provide sufficient conditions on the control and estimation horizons, and the amount of measurements, under which a line network gives maximal throughput under the proposed MPC-MHE controller. Third, we illustrate through simulations, that an appropriate extension of the MPC-MHE can maintain maximal throughput even under less measurements.

We conclude this section by stating key notations to be used throughout the work. For integers n_1 and $n_2 \geq n_1$, we let $[n_1 : n_2] := \{n_1, n_1 + 1, \ldots, n_2\}$. For brevity, $[1 : n_1]$ will be denoted compactly as $[n_1]$. We shall use $x(t_1 : t_2)$ to denote the sequence $\{x(t_1), x(t_1 + 1), \ldots, x(t_2)\}$. $\mathbb{R}_{\geq 0}$ will denote the set of non-negative reals. For vectors x and y, we shall let $x \leq y$ imply componentwise inequalities and let $x \prec y$ imply $x \leq y$ and $x \neq y$. For a vector x, we shall denote its i-th component either by x_i or $[x]_i$. All matrices are denoted by uppercase letters in boldface to differ from numbers denoted by uppercase letters. $\odot$ denotes the elementwise multiplication between vectors. $\mathbf{I}$ will denote the identity matrix whose dimension will be clear from the context. For $x \in \mathbb{R}$, we let $[x]^+ := \max\{x, 0\}$.

4.1 Model and Problem formulation

Consider a line freeway segment divided into I *cells* indexed by $i = 1, ..., I$ each with one on-ramp and one off-ramp; see Figure 1 for illustration. Extension to the case when some cells do not have on-ramp or off-ramp is straightforward and will not affect the results qualitatively. Let λ_i represent the exogenous traffic demand at ramp i. Let $x_i^m(t)$ and $x_i^r(t)$ denote the number of vehicles in cell i and on ramp i, respectively, at time t. Let $f_i(t)$ and $u_i(t)$ denote outflow from cell i and the inflow into cell i from its on-ramp, respectively, at time t. Therefore, u_i can be interpreted as a ramp-metering control at ramp i. Let $\beta_i \in (0, 1)$, denote the fraction of outflow from cell i that enters cell $i+1$ at time t; the rest of the $1 - \beta_i$ fraction exits through the off-ramp. Let $x^m(t) := \{x_i^m(t) : i \in [I]\}$,

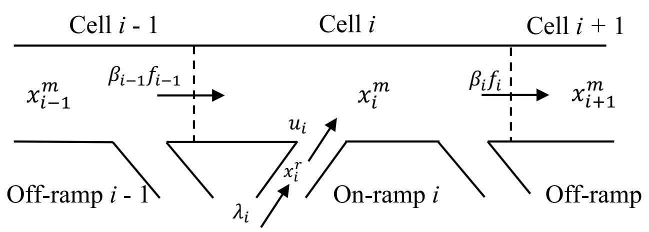

Figure 1: A line freeway segment.

$x^r(t) := \{x_i^r(t) : i \in [I]\}$, $u(t) := \{u_i(t) : i \in [I]\}$, and $\lambda := \{\lambda_i : i \in [I]\}$, be the compact notations. We use the Cell Transmission Model [15] with the triangular fundamental diagram to describe traffic flow dynamics:

$$\begin{aligned}
x_1^m(t+1) &= [x_1^m(t) - f_1(x^m(t)) + u_1(t)]^+ \\
x_i^m(t+1) &= [x_i^m(t) + \beta_{i-1} f_{i-1}(x^m(t)) - f_i(x^m(t)) + u_i(t)]^+, \quad i \in [2 : I] \\
x_i^r(t+1) &= x_i^r(t) - u_i(t) + \lambda_i, \quad i \in [I] \\
f_i(x^m(t)) &= \min\left\{ v_i x_i^m(t), \frac{w_{i+1}}{\beta_i}\left(\bar{x}_{i+1}^m - x_{i+1}^m(t)\right), C_i^{\max} \right\}, \quad i \in [I-1] \\
f_I(x^m(t)) &= \min\left\{ v_I x_I^m(t), C_I^{\max} \right\}
\end{aligned} \tag{1}$$

where v_i, w_i, $\bar{x}_i$ and $C_i^{\max}$ are parameters in the fundamental traffic diagram. We assume unbounded queue storage capacity on ramps, i.e., $x_i^r(t) \in [0, +\infty)$ for all $i \in [I]$. The ramp metering control $u_i(t)$ satisfies:

$$u_i(t) \in \left[0, \min\{\bar{u}_i, x_i^r(t) + \lambda_i\}\right], \quad i \in [I] \tag{2}$$

where $\bar{u}_i$ denotes the flow capacity of ramp i. The constraint in (2) ensures that the traffic state satisfies $x_i^m(t) \in [0, \bar{x}_i]$ for all $i \in [I]$, $t \geq 0$, under the dynamics in (1).

Let $\mathbf{R}$ be an $I \times I$ matrix such that $R_{i,i-1} = \beta_{i-1}$ for all $i \in [2 : I]$, and all other entries being zero. The control inputs designed in this project will ensure non-negativity of x^m. Under this implicit assumption, (1) can be compactly written as:

$$\begin{aligned} x^m(t+1) &= x^m(t) - (\mathbf{I} - \mathbf{R}) f(x^m(t)) + u(t) \\ x^r(t+1) &= x^r(t) - u(t) + \lambda \end{aligned} \tag{3}$$

where $f(x^m)$ is the compact notation representing the relationship between outflow and the number of vehicles in (1). Finally, let $x(t) = [x^m(t)\ x^r(t)]^T$ denote the state of the entire network.

In the uncontrolled case, i.e., when $u(t) \equiv \lambda$, mainline equilibrium flow is:[1]

$$f^{\text{eq}}(\lambda) = (\mathbf{I} - \mathbf{R})^{-1} \lambda \tag{4}$$

Let $x_i^{\text{crit}} := \dfrac{C_i^{\max}}{v_i}, i \in [I]$ be the *critical* queue lengths on the mainline cells. We say that a mainline cell i is uncongested if $x_i^m \leq x_i^{\text{crit}}$. It is shown in [16] that, for every λ satisfying (9), there exists a unique *uncongested* equilibrium state on the mainline. Let us denote this as $x^{\text{unc}}(\lambda)$, i.e., $f_i(x^{\text{unc}}(\lambda)) = v_i x_i^{\text{unc}}(\lambda)$ for all $i \in [I]$ (see Fig 2 for illustration).

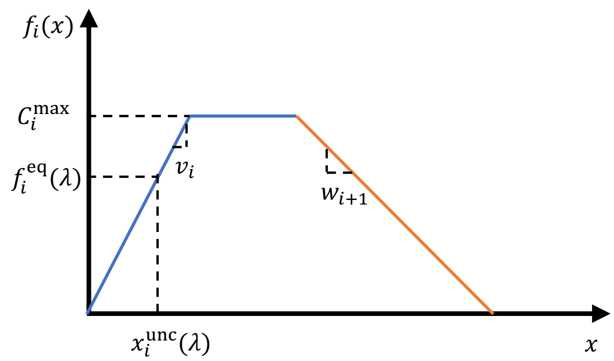

Figure 2: Illustrations of the parameters of the fundamental diagram

We consider the following output:

$$\underbrace{\begin{bmatrix} y^m(t) \\ y^r(t) \end{bmatrix}}_{y(t)} = \begin{bmatrix} \mathbf{C} & 0 \\ 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} x^m(t) \\ x^r(t) \end{bmatrix} \tag{5}$$

for an appropriate, not necessarily square, matrix $\mathbf{C}$. That is, we assume that we have perfect information about states of the ramps, but not necessarily for the mainline.

Example 1 Occupancy rate *is a commonly available measurement, e.g., through loop detectors [17], or from connected vehicles [4]. [18] shows that the occupancy rate is equal to* $\dfrac{\text{average vehicle length}}{\text{length of cell } i} x_i^m(t)$. *Therefore, when occupancy rate is available,* $\mathbf{C}$ *in* (5) *is a diagonal matrix, with zero diagonal entries corresponding to missing detectors on the corresponding cells.*

[1]Since all the elements in $\mathbf{R}$ are strictly less than one, and hence the spectral radius of $\mathbf{R}$ is less than one, the inverse $(\mathbf{I} - \mathbf{R})^{-1}$ exists and all of its entries are nonnegative.

It is of interest to design a control signal $\{u(t) : t \geq 0\}$ to minimize a linear cost $\sum_{t=0}^{\infty} l^\top x(t)$ subject to (2)-(3), for given non-negative coefficient vector l. Instead of solving this infinite horizon problem, the *model predictive control* (MPC) approach recursively solves a related finite horizon problem, which at time t is:

$$\sum_{s=t+1}^{t+T-1} l^\top x(s) + b^\top x(t+T) \tag{6}$$

subject to (2)-(3) and a *terminal constraint* $x(t+T) \in \mathcal{X}_f$, where T is the (forward) horizon, and the coefficient vector b in the *terminal cost* is non-negative. If $\{\hat{u}(t), \ldots, \hat{u}(t+T-1)\}$ denotes optimal solution to (6), then $u(t)$ is set to be equal to $\hat{u}(t)$, then (6) is re-solved at $t+1$ to similarly obtain $u(t+1)$, and so on.

In standard MPC, $x(t)$ is assumed to be known when solving (6). In this project, we are rather interested in the setting where, the controller has knowledge of the past measurements $y(t), y(t-1), \ldots$ when solving (6). A natural approach then is to augment MPC with an *estimation* component. We adopt the *moving horizon estimation* (MHE) approach, and accordingly pursue the joint MPC-MHE approach, e.g., see [14]. The next section elaborates on this approach.

4.2 The MPC-MHE Ramp Metering Controller

We start by introducing a few notations. For $t_2 \geq t_1$, let $\phi_{t_1}^{t_2}(x^\circ, u(t_1 : t_2-1))$ denote the state at time t_2 under (3) starting from $x(t_1) = x^\circ$ under control inputs $u(t_1 : t_2-1)$, and let $\phi_{t_1}^{t_2,m}(x^{\circ,m}, u(t_1 : t_2-1))$ denote the corresponding mainline state. For given input sequence $u(t_1 : t_2-1)$ and output sequence $y(t_1 : t_2)$, let the set of feasible initial conditions on the mainline be:

$$\begin{aligned}\mathcal{X}_{t_1:t_2}^{\circ,m} := \Big\{ x^{\circ,m} \in [0, \bar{x}] : &\text{the trajectory } \phi_{t_1}^{t_2}\left(\begin{bmatrix} x^{\circ,m} \\ x^r(t_1)\end{bmatrix}, u(t_1 : t_2-1)\right) \text{ satisfies (5) for } t \in [t_1, t_2], \\ &\phi_{t_1}^{t_2,m}\left(x^{\circ,m}, u(t_1 : t_2-1)\right) \leq \bar{x} \text{ for } s = t_1, \ldots, t_2 \Big\}\end{aligned} \tag{7}$$

and let the set of feasible control inputs over a future time horizon T be:

$$\mathcal{U}_{t_1:t_2}(T, \bar{x}^r) := \bigcap_{x^{\circ,m} \in \mathcal{X}_{t_1:t_2}^{\circ,m}} \tilde{\mathcal{U}}_{t_1:t_2}(T, \bar{x}^r; x^{\circ,m})$$

$$\begin{aligned}\tilde{\mathcal{U}}_{t_1:t_2}(T, \bar{x}^r; x^{\circ,m}) := \Big\{ &\hat{u}(t_2 : t_2+T-1) : \hat{u}_i(t) \in [0, \min\{x_i^r(t) + \lambda_i, \bar{u}_i\}], \ \forall i \in [I], \ t \in [t_2, t_2+T-1], \\ &\phi_{t_1}^{t}\left(\begin{bmatrix} x^{\circ,m} \\ x^r(t_1)\end{bmatrix}, u(t_1 : t_2-1), \hat{u}(t_2 : t-1)\right) \leq \bar{x}, \ \forall t \in [t_2, t_2+T-1], \\ &\phi_{t_1}^{t_2+T}\left(\begin{bmatrix} x^{\circ,m} \\ x^r(t_1)\end{bmatrix}, u(t_1 : t_2-1), \hat{u}(t_2 : t_2+T-1)\right) \in \mathcal{X}_f \Big\}\end{aligned} \tag{8}$$

where $\mathcal{X}_f := \{x : 0 \leq x^m \leq x^{\text{crit}}, \, 0 \leq x^r \leq \bar{x}^r\}$.

$\mathcal{U}_{t_1:t_2}(T, \bar{x}^r)$ is non-empty if λ is sufficiently small and if T is sufficiently large. It is natural to assume that λ is sufficiently small so that

$$f_i^{\text{eq}}(\lambda) \leq C_i^{\max}, \quad i \in [I] \tag{9}$$

i.e., the equilibrium flow $f^{\text{eq}}(\lambda)$ in the uncontrolled case does not exceed the flow capacity of the fundamental diagram on mainline cells.

Proposition 1 *If inflow λ satisfies (9) with strict inequalities, then, for every $x^{\circ,m} \in \mathcal{X}^{\circ,m}_{t_1:t_2}$ and $\bar{x}^r$, $\tilde{\mathcal{U}}_{t_1:t_2}(T, \bar{x}^r; x^{\circ,m})$ is non-empty for sufficiently large T.*

For brevity, we shall denote the set in (8) for the case when $\bar{x}^r = 0$ simply as $\mathcal{U}_{t_1:t_2}(T)$.

MPC-MHE controller augments control design over a forward horizon of length T with state estimation based on observation from the past horizon of length L. Specifically, at each time t, for given input sequence $u(t-L:t-1)$, output sequence $y(t-L:t)$, we solve the following min-max problem:

$$\min_{\hat{u}(t:t+T-1) \in \mathcal{U}_{t-L:t}(T)} \max_{x^{\circ,m} \in \mathcal{X}^{\circ,m}_{t-L:t}} J(x^{\circ,m}, \hat{u}(t:t+T-1)) := \sum_{s=t}^{t+T-1} l^\top \phi^s_{t-L} + b^\top \phi^{t+T}_{t-L} \tag{10}$$

where $\phi^s_{t-L} \equiv \phi^s_{t-L}\left(\begin{bmatrix} x^{\circ,m} \\ x^r(t-L) \end{bmatrix}, u(t-L:t-1), \hat{u}(t:s-1)\right)$, $s \in [t:t+T]$.

Let $(x^{\circ,m,*}, \hat{u}^*(t:t+T-1))$ denote a solution to (10). The control input for time t is chosen as $u(t) = \hat{u}^*(t)$. The process is then repeated at $t+1$ to find $u(t+1)$, and so on.

4.3 Input-to-State Stability of the MPC-MHE Controller

We now find sufficient conditions under which the MPC-MHE controller is stabilizing. Following [14], this relates to the existence of saddle-point for the min-max problem in (10). We introduce an assumption on the matrix $\mathbf{C}$ towards this purpose, which will also address the undesirable feature of the dependence on all feasible initial conditions $\mathcal{X}^{\circ,m}_{t-L:t}$ in (8).

Assumption 1 *$\mathbf{C}$ is a diagonal matrix such that for every nonzero element $c_{i,i}$, there exists at least one nonzero adjacent element $c_{i-1,i-1}$ or $c_{i+1,i+1}$, and for every zero element $c_{j,j}$, both $c_{j-1,j-1}$ and $c_{j+1,j+1}$ are nonzero.*

Proposition 2 *Let Assumption 1 be true. There exists $\hat{x}^{\circ,m} \in \mathcal{X}^{\circ,m}_{t_1:t_2}$ such that $\mathcal{U}_{t_1:t_2}(T) = \hat{\mathcal{U}}_{t_1:t_2}(T; \hat{x}^{\circ,m})$.*

Remark 1 *The $\mathbf{C}$ matrix in Example 1 is diagonal; it satisfies Assumption 1 if measurement is available from an appropriate combination of cells. Note that one needs measurements from at least 2/3 fraction of the cells for the assumption to be true.*

Theorem 1 *Let Assumption 1 be true, T be sufficiently large, and the inflow λ satisfy (9) with strict inequalities. Then, for every $t \geq L$, past control input sequence $u(t-L:t-1)$, and past measured output sequence $y(t-L:t)$, there exist $x^{\circ,m,*} \in \mathcal{X}^m$ and $\hat{u}^*(t:t+T-1)$ such that*

$$\begin{aligned} J(x^{\circ,m,*}, \hat{u}^*(t:t+T-1)) &= \min_{\hat{u}(t:t+T-1) \in \mathcal{U}_{t-L:t}(T)} J(x^{\circ,m,*}, \hat{u}(t:t+T-1)) \\ &= \max_{x^{\circ,m} \in \mathcal{X}^{\circ,m}_{t-L:t}} J(x^{\circ,m}, \hat{u}^*(t:t+T-1)) \end{aligned} \tag{11}$$

Moreover, the initial condition $x^{\circ,m,*}$ is the unique maximal element in $\mathcal{X}^{\circ,m}_{t-L:t}$.

Remark 2 1. *Theorem 1 suggests that to find a pair $(x^{\circ,m*}, \hat{u}^*(t:t+T-1))$, one can first find the maximal element $x^{\circ,m*}$ in $\mathcal{X}^{\circ,m}$ and then solve the minimization problem in (11) for $\hat{u}^*(t:t+T-1)$.*

2. If conditions of Theorem 1 hold true then one can write

$$J(x^{\circ,m,*}, \hat{u}^*(t:t+T-1)) = \min_{\hat{u}(t:t+T-1)\in\mathcal{U}_{t-L:t}(T)} V(x^{m,*}(t), \hat{u}(t:t+T-1)) =: V^*(x^{m,*}(t))$$

where $x^{m,*}(t) = \phi_{t-L}^{t,m}(x^{\circ,m,*}, u(t-L:t-1))$*, in terms of a* cost-to-go *function* V*.*

We now state the result on the *input-to-state stability* of the closed-loop system consisting of (3), (5) in feedback with the MPC-MHE controller in (11), with the exogenous inflow λ interpreted as the *input* in this context. In order to achieve this, we need additional conditions on the system, the backward horizon L, and the cost coefficients.

Theorem 2 *Let Assumption 1 be true,* T *be sufficiently large, and the inflow* λ *satisfy* (9) *strictly. Furthermore, let the following be true: (i) for all* $u(t-L-1:t-1)$*,* $y(t-L-1:t)$ *and* $x^r(t-L-1:t)$*,* $t \geq L+1$*:*

$$\forall\, x^{\circ,m} \in \mathcal{X}_{t-L:t}^{\circ,m},\ \exists\, \tilde{x}^{\circ,m} \ \ s.t.\ \ \phi_{t-L-1}^{t-L,m}(\tilde{x}^{\circ,m}, u(t-L-1)) = x^{\circ,m} \quad \& \quad y(t-L-1) = \mathbf{C}\begin{bmatrix} \tilde{x}^{\circ,m} \\ x^r(t-L-1) \end{bmatrix} \tag{12}$$

and, (ii) for every λ *satisfying* (9) *strictly, there exists a control* u *that satisfies* (2) *and a* $\eta \geq 0$ *such that*

$$\begin{aligned} & b^\top \phi_t^{t+1}(x,u) - b^\top x \leq -l^\top x + \eta^\top \lambda \\ & \phi_t^{t+1}(x,u) \in \mathcal{X}_f, \qquad \forall\, x \in \mathcal{X}_f \end{aligned} \tag{13}$$

Then, the closed-loop system consisting of (3), (5) *in feedback with the MPC-MHE controller in* (11) *is* input-to-state stable *with respect to* λ*, i.e.,*

$$\|x(t)\|_1 \leq \beta(\|\hat{x}(L)\|_1, t) + \gamma(\|\lambda\|_1), \quad \forall\, t \geq L, \quad x(0) \in \mathcal{X} \tag{14}$$

where $\beta(k,t) := \left(1 - \frac{a_1}{a_2}\right)^t \frac{3a_2 k}{a_1}$ *and* $\gamma(k) := \frac{3a_2\eta_{\max}k}{a_1^2}$ *with* $\eta_{\max} = \max_i \eta_i$*,* $a_1 = \min_i l_i$ *and* $a_2 = T\max_{i\in[I]} \frac{\bar{x}_i}{x_i^{crit}} \cdot \max\{l_i, b_i\}$*.*

Remark 3 *It is straightforward to see that* $\tilde{x}^{\circ,m}$ *which satisfies* (12) *belongs to* $\mathcal{X}_{t-L-1:t-1}^{\circ,m}$*.*

Example 2 *A combination of* u*,* l *and* b *is as follows. Pick* $u = \lambda$*,* l *to be arbitrary in* $\mathbb{R}^I_{\geq 0}$*, and* b *such that* $b_i = [(\mathbf{I}-\mathbf{R})^{-\top} l]_i / v_i$*,* $i \in [I]$*. Recall that for every* $x \in \mathcal{X}_f$*, we have* $x^r = 0$ *and* $f_i(x^m) = v_i x_i^m$*. Therefore,* $b^\top(\phi(x,u) - x) = b^\top(\lambda - (\mathbf{I}-\mathbf{R})f(x^m)) \leq -l^\top x + \eta^\top\lambda$*, where the inequality follows from* $b_i = [(\mathbf{I}-\mathbf{R})^{-\top} l]_i / v_i$*,* $i \in [I]$*. This establishes the first condition in* (13)*. The second condition follows from the proof of Proposition 1 in Appendix 4.6, where we show positive invariance of* $\mathcal{X}_f$ *under* $u = \lambda$*.*

4.4 Simulations

Consider a line network consisting of ten cells. Similar to [16], we assume homogeneous cells, with parameters $v_i = 0.5$veh/time step, $w_i = \frac{0.5}{3}$veh/time step, $\bar{x}_i = 160$ veh/cell, $C_i^{\max} = 20$ veh/time step. [$\mathbf{R}$ =?]. Performance of the MPC-MHE controller is evaluated in terms of throughput, for different $\mathbf{C}$; throughput is defined as the maximal element in $(\mathbf{I}-\mathbf{R})^{-1}\lambda$ as throughput.

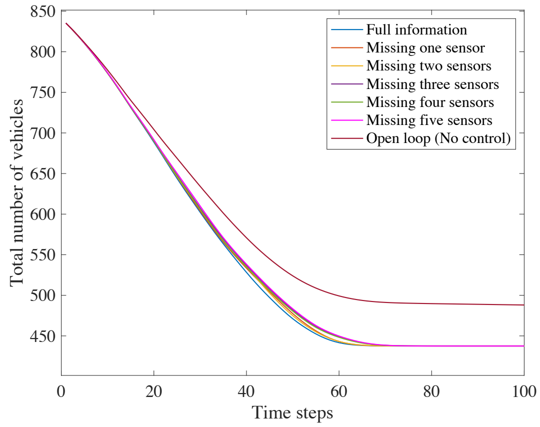

Figure 3: Traffic evolution under maximal demand with various numbers of missing sensors

We investigate the ability of the traffic flow under MPC-MHE control to maintain throughput set to 19.9 veh/time step that is closed to $C^{\max}$. As shown in Figure 3, with missing sensors, the controller is still able to maintain the throughput. For performance in terms of the total number of vehicles, full information provides a lower bound on the performance. As the number of missing sensors increases, the performance becomes worse but still significantly better than no control case.

4.5 Technical Results

In this section, we collect technical results to be used later in the proofs.

The following *strictly monotone* property of the dynamics in (3) is proven in [16].

Lemma 1 *For any control sequence $u(t_1 : t_2 - 1)$ and mainstream states $x^{\circ,m,1} \prec x^{\circ,m,2}$, we have $\phi_{t_1}^{t_2,m}(x^{\circ,m,1}, u(t_1 : t_2 - 1)) \prec \phi_{t_1}^{t_2,m}(x^{\circ,m,2}, u(t_1 : t_2 - 1))$ for all $t_2 \geq t_1$.*

Definition 1 *A set X is said to be closed with respect to componentwise maximization if the following implication holds: $x^1, x^2 \in X \Longrightarrow \max\{x^1, x^2\} \in X$, where* max *is elementwise maximum.*

The following result is straightforward, whose proof can be found in [19].

Lemma 2 *Let set X be compact and closed with respect to componentwise maximization. Then, there exists a unique $\hat{x} \in X$ such that $\hat{x} \geq x$ for all $x \in X$.*

We call such a element $\hat{x}$ the *maximal* element in the set X.

4.6 Proof of Proposition 1

It is sufficient to provide proof for the special case when $\bar{x}^r = 0$. Following Proposition 2 and Lemma 1, it suffices to show that the set $\tilde{\mathcal{U}}_{t_1:t_2}(T, 0; \hat{x}^{\circ,m})$ is not empty. Consider the following control policy:

$$u_i(t) = \begin{cases} 0, & \exists\, j \in [I] \text{ s.t. } x_j^m(t) > x_j^{\text{crit}} \\ \min\{x_1^r(t) + \lambda_1, v_1\, x_1^{\text{crit}}\}, & i = 1, \quad \& \quad x^m(t) \leq x^{\text{crit}} \\ \min\{x_i^r(t) + \lambda_i, v_i x_i^{\text{crit}} - \beta_{i-1} v_{i-1} x_{i-1}^m(t)\}, & i \in [2:I], \quad \& \quad x^m(t) \leq x^{\text{crit}} \end{cases} \tag{15}$$

This control policy first steers the mainline to uncongested regime, i.e., to $[0, x^{\text{crit}}]$, in finite time by stopping inflow into the mainline altogether. Therefore, without loss of generality, one can assume that the initial condition is in $[0, x^{\text{crit}}]$. From such an initial condition, we now show that, under the control policy in (15), $[0, x^{\text{crit}}]$ is positively invariant. For $i = 1$, (15) implies that $u_1(t) \leq v_1 x_1^{\text{crit}}$. Combining this with $x_1^{\text{crit}} = x_1^{\text{crit}} + v_1 x_1^{\text{crit}} - v_1 x_1^{\text{crit}}$, $x_1^m(0) \leq x_1^{\text{crit}}$, and Lemma 1, gives $x_1^m(1) \leq x_1^{\text{crit}}$. For $i \geq 2$, (15) implies $u(0) \leq v_i x_i^{\text{crit}} - \beta_{i-1} v_{i-1} x_{i-1}^m(0)$. Therefore, $x_i^{\text{crit}} \geq x_i^{\text{crit}} + \beta_{i-1} v_{i-1} x_{i-1}^m(0) - v_i x_i^{\text{crit}} + u_i(0) \geq x_i^m(0) + \beta_{i-1} v_{i-1} x_{i-1}^m(0) - v_i x_i^m + u_i(0) = x_i^m(1)$.

We now show that on-ramp queue lengths are steered to zero in finite time, and that the on-ramp queue lengths remain zero thereafter. Let $\mathcal{L}(t) := \{i \in [I] : x_i^r(t) \geq v_i x_i^{\text{crit}} - \beta_{i-1} v_{i-1} x_{i-1}^m(t) - \lambda_i\}$. For $i \neq \mathcal{L}(t)$, $x_i^r(t+1) = x_i^r(t) + \lambda_i - u_i(t) = x_i^r(t) + \lambda_i - (x_i^r(t) + \lambda_i) \equiv 0$, i.e., once an on-ramp is outside $\mathcal{L}(t)$ it stays there and its queue length reaches zero and stays there in at most one time step.

For $i \in \mathcal{L}(t)$, $u_i(t) = v_i x_i^{\text{crit}} - \beta_{i-1} v_{i-1} x_{i-1}^m(t) \geq v_i x_i^{\text{crit}} - \beta_{i-1} v_{i-1} x_{i-1}^{\text{crit}}(t) = [(\mathbf{I} - \mathbf{R}) f(x^{\text{crit}})]_i$. Combining this with $\lambda = (\mathbf{I} - \mathbf{R}) x^{\text{unc}}(\lambda)$, (1) gives:

$$
\begin{aligned}
\sum_{i \in \mathcal{L}(t)} x_i^r(t+1) &\leq \sum_{i \in \mathcal{L}(t)} x_i^r(t) + \mathbb{1}^\top \left(\mathbf{I} - \mathbf{R}_{\mathcal{L}(t)}\right) \left(f_{\mathcal{L}(t)}(x^{\text{unc}}(\lambda)) - f_{\mathcal{L}(t)}(x^{\text{crit}})\right) \\
&\leq \sum_{i \in \mathcal{L}(t)} x_i^r(t) - |\mathcal{L}(t)| \min_{i \in \mathcal{L}(t)} [\mathbb{1}^\top (\mathbf{I} - \mathbf{R})]_i \cdot [\left(f(x^{\text{crit}}) - f(x^{\text{unc}}(\lambda))\right)]_i \qquad (16)
\end{aligned}
$$

where $\mathbb{1}$ is vector of all ones, $\mathbf{R}_{\mathcal{L}(t)}$ is the sub-matrix of $\mathbf{R}$ corresponding to the rows/columns in $\mathcal{L}(t)$; similarly $f_{\mathcal{L}(t)}(x^{\text{unc}}(\lambda))$ and $f_{\mathcal{L}(t)}(x^{\text{crit}})$ are the sub-vectors of $f(x^{\text{unc}}(\lambda))$ and $f(x^{\text{crit}})$, respectively, corresponding to entries in $\mathcal{L}(t)$. The entries of $f(x^{\text{unc}}(\lambda)) - f(x^{\text{crit}})$ are all negative, and the entries of $\mathbf{R}_{\mathcal{L}(t)}$ are all strictly less than one. (16) implies that the sum of queue lengths on ramps in $\mathcal{L}(t)$ is strictly decreasing and the rate of decrease is bounded away from zero. Therefore, $\mathcal{L}(t)$ becomes empty in finite time. Combined with the earlier conclusion that queue lengths on ramps not in $\mathcal{L}(t)$ go to zero and stay at zero in at most one time step gives the desired result.

4.7 Proof of Proposition 2

The proposition follows once we show that $\hat{x}^{\circ,m}$ is the maximal element of $\mathcal{X}^{\circ,m}$. This is because $\tilde{\mathcal{U}}_{t_1:t_2}(T; \hat{x}^{\circ,m}) \subseteq \mathcal{U}_{t_1:t_2}(T)$ then follows from the strict monotonicity property in Lemma 1.

$\mathcal{X}_{t_1:t_2}^{\circ,m}$ is compact by definition in (7). Therefore, following Lemma 2, we just need to show that $\mathcal{X}_{t_1:t_2}^{\circ,m}$ is closed with respect to componentwise maximization. Consider $x^{\circ,m,1} \in \mathcal{X}_{t_1:t_2}^{\circ,m}$, $x^{\circ,m,2} \in \mathcal{X}_{t_1:t_2}^{\circ,m}$, $x^{\circ,m,1} \neq x^{\circ,m,2}$. Let $x^{\circ,m} := \max\{x^{\circ,m,1}, x^{\circ,m,2}\}$ be the elementwise maximum. We first show by induction on t that the trajectory $\phi_{t_1}^t \left(\begin{bmatrix} x^{\circ,m} \\ x^r(t_1) \end{bmatrix}, u(t_1 : t-1) \right) \equiv x^m(t)$ satisfies (5) for $t = t_1, \ldots, t_2$. Let the notations $x^{m,1}(t)$ and $x^{m,2}(t)$ be defined similarly.

For a cell i with $c_{i,i} \neq 0$, we have $y_i(t_1) = c_{i,i} x_i^{m,1}(t_1) = c_{i,i} x_i^{m,2}(t_1)$, giving $x_i^{m,1}(t_1) = x_i^{m,2}(t_1) = x^m(t_1)$, where the last equality follows from the definition of x^m. Therefore, (1) gives:

$$
\begin{aligned}
x_i^m(t_1+1) = &\, x_i^m(t_1) + \min\{\beta_{i-1} v_{i-1} x_{i-1}^m(t_1), w_i(\bar{x}_i - x_i^m(t_1)), \beta_{i-1} C_{i-1}^{\max}\} \\
&- \min\{v_i x_i^m(t), \frac{w_{i+1}}{\beta_i}(\bar{x}_{i+1} - x_{i+1}^m(t_1)), C_i^{\max}\} + u_i(t_1)
\end{aligned}
$$

Since $c_{i,i} x_i^{m,1}(t_1+1) = c_{i,i} x_i^{m,2}(t_1+1)$, we have $x_i^{m,1}(t_1+1) = x_i^{m,2}(t_1+1)$ and therefore, following (1):

$$\min\{\beta_{i-1}v_{i-1}x_{i-1}^{m,1}(t_1), w_i(\bar{x}_i - x_i^{m,1}(t_1)), \beta_{i-1}C_{i-1}^{\max}\} - \min\{v_i x_i^{m,1}(t_1), \frac{w_{i+1}}{\beta_i}(\bar{x}_{i+1} - x_{i+1}^{m,1}(t_1)), C_i^{\max}\}$$
$$= \min\{\beta_{i-1}v_{i-1}x_{i-1}^{m,2}(t_1), w_i(\bar{x}_i - x_i^{m,2}(t_1)), \beta_{i-1}C_{i-1}^{\max}\} - \min\{v_i x_i^{m,2}(t_1), \frac{w_{i+1}}{\beta_i}(\bar{x}_{i+1} - x_{i+1}^{m,2}(t_1)), C_i^{\max}\} \tag{17}$$

Following Assumption 1, let $c_{i-1,i-1} \neq 0$, i.e., $x_{i-1}^{m,1}(t_1) = x_{i-1}^{m,2}(t_1)$; the alternative case of $c_{i+1,i+1} \neq 0$, i.e., $x_{i+1}^{m,1}(t_1) = x_{i+1}^{m,2}(t_1)$ can be handled similarly. Using $x_i^{m,1}(t_1) = x_i^{m,2}(t_1)$ in (17), we have that

$$\begin{aligned}\min\{v_i x_i^{m,1}(t_1), \frac{w_{i+1}}{\beta_i}(\bar{x}_{i+1} - x_{i+1}^{m,1}(t_1)), C_i^{\max}\} &= \min\{v_i x_i^{m,2}(t_1), \frac{w_{i+1}}{\beta_i}(\bar{x}_{i+1} - x_{i+1}^{m,2}(t_1)), C_i^{\max}\} \\ &= \min\{v_i x_i^{m}(t_1), \frac{w_{i+1}}{\beta_i}(\bar{x}_{i+1} - x_{i+1}^{m}(t_1)), C_i^{\max}\}\end{aligned}$$

where the last equality again follows from the definition of x^m, since $x_i^{m,1}(t_1) = x_i^{m,2}(t_1) = x^m(t_1)$ and $\frac{w_{i+1}}{\beta_i}(\bar{x}_{i+1} - x_{i+1}^{m}(t_1))$ is the minimum of $\frac{w_{i+1}}{\beta_i}(\bar{x}_{i+1} - x_{i+1}^{m,1}(t_1))$ and $\frac{w_{i+1}}{\beta_i}(\bar{x}_{i+1} - x_{i+1}^{m,2}(t_1))$. Therefore, (17) is equal to $\min\{\beta_{i-1}v_{i-1}x_{i-1}^{m}(t_1), x_i(\bar{x}_i - x_i^{m}(t_1)), \beta_{i-1}C_{i-1}^{\max}\} - \min\{v_i x_i^{m}(t_1), \frac{w_{i+1}}{\beta_i}(\bar{x}_{i+1} - x_{i+1}^{m}(t_1)), C_i^{\max}\}$. Following (1), this implies that $x_i^{m,1}(t_1+1) = x_i^{m,2}(t_1+1) = \phi_{t_1}^{t_1+1}(x_i^m(t_1), u(t_1)) =: x_i^m(t_1+1)$.

For a cell i with $c_{i,i} = 0$, following Assumption 1, we have $c_{i+1,i+1} \neq 0$ and $c_{i-1,i-1} \neq 0$. Following similar analysis as above, these respectively imply that $x_{i+1}^{m,1}(t_1) = x_{i+1}^{m,2}(t_1) = x_{i+1}^{m}(t_1)$ and $x_{i-1}^{m,1}(t_1) = x_{i-1}^{m,2}(t_1) = x_{i-1}^{m}(t_1)$, and hence $f_i(x^{m,1}(t_1)) = f_i(x^{m,2}(t_1)) = f_i(x^m(t_1))$ and $f_{i-1}(x^{m,1}(t_1)) = f_{i-1}(x^{m,2}(t_1)) = f_{i-1}(x^m(t_1))$ respectively. Therefore, $x_i^m(t_1+1) = x_i^m(t_1) + \beta_{i-1}f_{i-1}(x^m(t_1)) - f_i(x^m(t_1)) = \max\{x_i^{m,1}(t_1), x_i^{m,2}(t_1)\} + \beta_{i-1}f_{i-1}(x^m(t_1)) - f_i(x^m(t_1)) = \max\{x_i^{m,1}(t_1+1), x_i^{m,2}(t_1+1)\} \leq \bar{x}_i$.

4.8 Proof of Theorem 1

For every input sequence $\hat{u}(t:t+T-1) \in \mathcal{U}_{t-L:t}(T)$, the maximization problem in (10):

$$\max_{x^{\circ,m} \in \mathcal{X}_{t-L:t}^{\circ,m}} J(x^{\circ,m}, \hat{u}(t:t+T-1)) \tag{18}$$

admits a unique solution $x^{\circ,m,*}$ which is the maximal element of $\mathcal{X}_{t-L:t}^{\circ,m}$. This is because every other $x^{\circ,m} \in \mathcal{X}_{t-L:t}^{\circ,m}$ satisfies that $x^{\circ,m} \prec x^{\circ,m,*}$, which implies that $\phi_{t-L}^{s,m}(x^{\circ,m}, u(t-L:t-1), \hat{u}(t:s-1)) \prec \phi_{t-L}^{s,m}(x^{\circ,m,*}, u(t-L:t-1), \hat{u}(t:s-1))$ for all $s \in [t-L:t+T]$ according to Lemma 1. Since l and b are positive, this then implies $J(x^{\circ,m}, \hat{u}(t:t+T-1)) < J(x^{\circ,m,*}, \hat{u}(t:t+T-1))$.

Let $\hat{u}^*(t:t+T-1)$ be an optimal solution to the minimization problem in (10):

$$\min_{\hat{u}(t:t+T-1) \in \mathcal{U}_{t-L:t}(T)} J(x^{\circ,m,*}, \hat{u}(t:t+T-1))$$

Therefore, $J^* = J(x^{\circ,m,*}, \hat{u}^*(t:t+T-1))$. Since $x^{\circ,m,*}$ is the unique optimal solution in (18),

$$J(x^{\circ,m,*}, \hat{u}^*(t:t+T-1)) = \max_{x^{\circ,m} \in \mathcal{X}^m} J(x^{\circ,m}, \hat{u}^*(t:t+T-1))$$

4.9 Proof of Theorem 2

The proof is in two steps:

1. Consider the trajectory $\left\{\hat{x}(t) := \begin{bmatrix} x^{\circ,m,*}(t) \\ x^r(t) \end{bmatrix} : t \geq 0\right\}$ constructed from the solution to (10). This $\hat{x}(t)$ is a trajectory of the *nominal* closed-loop system consisting of (3) in feedback with the *state feedback* MPC controller corresponding to the minimization problem in (11).

2. The value function in (11) possesses a dissipativity-like property and serves as an input-to-state Lyapunov function for the nominal closed-loop system. The input-to-state stability of this nominal system then implies input-to-state stability of the closed-loop system consisting of (3), (5) in feedback with the output-feedback MPC-MHE controller in (10).

We now prove these two claims.

Step 1: Theorem 1 implies that there exists a unique maximal element, say $x^{\circ,m,*}(t+1)$, in $\mathcal{X}^{\circ,m}_{t+1:t+L+1}$. Moreover, (12) implies that $\tilde{x}^{\circ,m} \in \mathcal{X}^{\circ,m}_{t:t+L}$ (cf. Remark 3). We now show that in fact $\tilde{x}^{\circ,m}$ is the unique maximal element $x^{\circ,m,*}(t)$ of $\mathcal{X}^{\circ,m}_{t:t+L}$. Suppose not. Therefore, $\tilde{x}^{\circ,m} \prec x^{\circ,m,*}(t)$. Lemma 1 then implies that $x^{\circ,m,*}(t+1) = \phi_t^{t+1}(\tilde{x}^{\circ,m}, u(t)) \prec \phi_t^{t+1}(x^{\circ,m,*}(t), u(t))$, which contradicts that $x^{\circ,m,*}(t+1)$ is the unique maximal element in $\mathcal{X}^{\circ,m}_{t+1:t+L+1}$. Therefore, $\tilde{x}^{\circ,m} = x^{\circ,m,*}(t)$ and $x^{\circ,m,*}(t+1) = \phi_t^{t+1,m}(x^{\circ,m,*}(t), u(t))$. One can continue along these lines to show by induction that $x^{\circ,m,*}(t) = \phi_{t-L}^{t,m}(x^{\circ,m,*}(t-L), u(t-L:t-1))$.

Since $x^{\circ,m,*}(t-L)$ is the unique maximizer in (11), $u(t)$ can be equivalently obtained by solving the minimization in (11) for state $x^{\circ,m,*}(t)$. In other words, the sequence $\hat{x}(t) := \begin{bmatrix} x^{\circ,m,*}(t) \\ x^r(t) \end{bmatrix}$, $t \geq 0$, can be interpreted as a trajectory of the closed-loop system consisting of (3) in feedback with state-feedback MPC controller defined by the minimization problem in (11).

Step 2: We now analyze input-to-state stability of the *nominal* closed-loop system. Let $\hat{x}(s)$, $s \in \{t+2, \ldots, t+T+1\}$ be the state values starting from $\hat{x}(t+1)$ given control sequence $\hat{u}(t+1:s-1)$. (13) implies that there exists $\tilde{u}(t+T)$ such that

$$b^\top \phi_{t+T}^{t+T+1}(\hat{x}(t+T), \tilde{u}(t+T)) - b^\top \hat{x}(t+T) \leq -l^\top \hat{x}(t+T) + \eta^\top \tilde{u}(t+T), \quad \forall\, \hat{x}(t+T) \in \mathcal{X}_f \tag{19}$$

Recall from Remark 2 that there exist $\hat{u}^*(t:t+T-1)$ and $\hat{u}^*(t+1:t+T)$ such that, respectively,

$$\begin{aligned} V^*(\hat{x}(t)) &= V(\hat{x}(t), \hat{u}^*(t:t+T-1)) \\ V^*(\hat{x}(t+1)) &= V(\hat{x}(t+1), \hat{u}^*(t+1:t+T)) \leq V(\hat{x}(t+1), \hat{u}^*(t+1:t+T-1), \tilde{u}(t+T)) \end{aligned} \tag{20}$$

where the last inequality follows from the feasibility of $\{\hat{u}^*(t+1:t+T-1), \tilde{u}(t+T)\}$. This is because $\hat{u}^*(t:t+T-1) \in \mathcal{U}_{t-L:t-1}(T)$ and therefore $\hat{x}(t+T) \in \mathcal{X}_f$; $\hat{x}(t+T+1) \in \mathcal{X}_f$ then follows from (13). (20) implies

$$\begin{aligned} V^*(\hat{x}(t+1)) - V^*(\hat{x}(t)) &\leq V(\hat{x}(t+1), \hat{u}^*(t+1:t+T-1), \tilde{u}(t+T)) - V(\hat{x}(t), \hat{u}^*(t:t+T-1)) \\ &\leq b^\top \phi(\hat{x}(t+T), \tilde{u}(t+T)) - b^\top \hat{x}(t+T) + l^\top \hat{x}(t+T) - l^\top \hat{x}(t) \\ &\leq -l^\top \hat{x}(t) + \eta^\top \lambda \end{aligned}$$

where the last inequality follows from (19). [20, Theorem 2.5] then implies that the nominal closed-loop system is input-to-state stable with respect to λ, i.e., (14) holds true.

Recall that the state of the nominal system, $\hat{x}(t)$ is the maximal element in $\mathcal{X}^{\circ,m}_{t-L:t}$ which also contains the state $x(t)$ of the actual system. That is, $\hat{x}(t) \geq x(t)$ for all $t \geq 0$. Moreover, it is straightforward to see that the MPC-MHE controller generates the same control signal as the state-feedback MPC controller. Therefore, Lemma 1 implies that the nominal closed-loop system is always an upper bound for the closed-loop system with the MPC-MHE controller, and hence (14) is true.

5 Conclusion and Future Work

In this project, we adopted the MPC-MHE framework to design an output feedback ramp metering control. For a line network modeled by the Cell Transmission Model, we provided sufficient conditions for input-to-state stability. There are several avenues for further research. It will be interesting to extend the results to general network configuration, other output models, general traffic flow models, and to other estimation techniques such as particle filtering and extended Kalman filter, as well as to estimation techniques specifically developed for traffic flow models, e.g., see [21–23]. This work could be thought of as a first step towards data-driven feedback traffic flow control with provable guarantees. Therefore, a natural next step could also be to study the model-free setting, e.g., in the spirit of [24]. Finally, it would be of interest to explore spatial sparsity of the dynamics to investigate sparsity in the structure of output feedback controller, along the lines of our recent work on MPC-based traffic flow control [25].

6 Implementation

Not applicable.

7 References

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8 Data Management Plan

Products of Research

No new data was collected for this project. Only simulation data was generated.

Data Format and Consent

All the simulations were done in Matlab.

Data Access and Sharing

The input/output data for the simulations in this report is available at `https://viterbi-web.usc.edu/~ksavla/code.html`.

Reuse and Redistribution

The data can be reused freely for non-commercial purposes. Its usage, in original or after modification, in publications is to be done with due acknowledgement to the authors of this report and by citation of relevant publications by the authors.