Dynamic Incentive Design for Transportation Systems with Unknown Value of Time

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About the Pacific Southwest Region University Transportation Center

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The Pacific Southwest Region UTC conducts an integrated, multidisciplinary program of research, education and technology transfer aimed at *improving the mobility of people and goods throughout the region.* Our program is organized around four themes: 1) technology to address transportation problems and improve mobility; 2) improving mobility for vulnerable populations; 3) Improving resilience and protecting the environment; and 4) managing mobility in high growth areas.

U.S. Department of Transportation (USDOT) Disclaimer

Disclosure

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Abstract

Congestion pricing is a common technique to steer the user choices towards a socially optimal profile. Determination of pricing requires knowledge of travelers' values of time (VOT). Currently, most methods for estimating VOT analyze certain groups of people, using data collected from surveys. We consider the problem of adaptive congestion pricing to learn unknown value of time and unknown coefficients of link latency functions, which are collectively referred to as the parameters of routing game. The input for each trial is pricing for each link and the output is the corresponding Nash flow. The input for a trial is allowed to depend on input-output from all previous trials. For polynomial link latency functions, we provide a necessary condition for unique identification of the parameters and provide an adaptive pricing policy which uniquely identifies the parameters in finite trials. This naturally translates into finite trial guarantee for adaptive pricing to minimize social cost. We implemented our adaptive pricing policy in traffic assignment human subject experiments and find that the estimated value of time is lower than the one estimated by a known adaptive stated preference method, but the corresponding marginal pricing is quite effective in inducing social equilibrium link flows.

Dynamic Incentive Design for Transportation Systems with Unknown Value of Time

Executive Summary

Congestion pricing is a common technique to steer the user choices towards a socially optimal profile. Determination of pricing requires knowledge of travelers' values of time (VOT). Currently, most methods for estimating VOT analyze certain groups of people, using data collected from surveys. These traditional methods typically require significant time and manpower to collect data, and the results are only applicable to specific groups, lacking accessibility and timeliness.

We consider the problem of adaptive congestion pricing to learn unknown value of time and unknown coefficients of link latency functions, which are collectively referred to as the parameters of routing game. The input for each trial is pricing for each link and the output is the corresponding Nash flow. The input for a trial is allowed to depend on input-output from all previous trials. For polynomial link latency functions, we provide a necessary condition for unique identification of the parameters and provide an adaptive pricing policy which uniquely identifies the parameters in finite trials. The pricing policy consists of multiple phases which adaptively identifies in finite trials a hypercube in the space of link prices whose Nash flow is supported on all links. The Nash flow condition can then be written as linear equations in unknown parameters. We then identify sufficient conditions on the number of samples from this hypercube to unknown parameters for polynomial link latency functions. This naturally translates into finite trial guarantee for adaptive pricing to minimize social cost.

We implemented our adaptive pricing policy in traffic assignment human subject experiment and compared it against a semi-adaptive boundary-value stated preference method. The experiment consists of two phases, both conducted on a computer. The first phase is based on the SABVSP survey method. After participants complete the survey, the individual VOT data derived from the survey is recorded. The second phase is traffic assignment over 3 links. It further consists of two parts. Part 1 implements our adaptive pricing policy, while part 2 implements marginal pricing based on the VOT estimated from part 1. In both the parts, the participants are instructed and incentivized to choose Nash flow with respect to the corresponding price as their traffic assignment. After both phases, participants are asked to fill out a questionnaire aimed at understanding their decision-making strategy. Experiment data analysis suggests that the value of time estimated using our adaptive pricing method is lower than the adaptive stated preference method, but the corresponding marginal pricing is quite effective in inducing social equilibrium link flows.

1 Introduction

It is well-known that, in general, user equilibria in traffic systems do not minimize social cost, e.g., average travel time of all travelers. Congestion pricing is a common technique to steer the user choices towards a socially optimal profile. Determination of pricing requires knowledge of travelers' values of time (VOT). VOT is pivotal for transportation planning and policy-making, guides the economic trade-offs that travelers make between time and cost. VOT is the cost of marginal changes in time spent traveling. In essence, it can be seen as the amount the traveler would be willing to pay in order to save time, or the amount they would accept compensation for lost time. Currently, most methods for estimating the VOT are primarily used to analyze certain groups of people, using data collected from Revealed Preference (RP) or Stated Preference (SP) surveys. These surveys collect data by observing real travel choices or by designing hypothetical scenarios, thus providing information on the trade-offs travelers make between time and cost. After collecting this data, logit models, such as the multinomial logit or mixed logit models, are typically used for statistical analysis [1]. Those traditional methods typically require significant time and manpower to collect data, and the results are only applicable to specific groups, lacking accessibility and timeliness.

We consider routing game setup between a single origin-destination along parallel links, with unknown value of time and unknown coefficients of link latency functions, which are collectively referred to as the *parameters* of routing game. We are interested in designing adaptive congestion pricing to determine these unknown parameters. The input for each trial is pricing for each link and the output is the corresponding Nash flow. The input for a trial is allowed to depend on input-output from all previous trials. The Nash flow for each trial can be described by variational inequalities. Parameter estimation for such settings through inverse optimization has been considered, e.g., in [2]. A robust generalization of this approach which particularly allows for the possibility of incomplete measurements is considered in [3]. An online extension of inverse optimization approaches, for limited setting, has been proposed recently, e.g., see [4]. This and other online versions typically assume that the input is generated either by an exogenous process or by an adversarial process. In this project, on the other hand, we are interested in designing input, i.e., prices, in an online manner so as to achieve a desired objective. Some recent work [5] partially addresses this problem for a special class. Finally, this work is to be contrasted with robust approaches to congestion pricing when the value of time is unknown, e.g., see [6].

We provide a multi-phase adaptive pricing policy which adaptively identifies in finite trials a hypercube in the space of link prices whose Nash flow is supported on all links. The Nash flow condition can then be written as linear equations in unknown parameters. We then identify sufficient conditions on the number of samples from this hypercube to unknown parameters for polynomial link latency functions. This naturally translates into finite trial guarantee for adaptive pricing to minimize social cost.

We implemented our adaptive pricing policy in traffic assignment human subject experiments, and compared it against a semi-adaptive boundary-value stated preference (SABVSP) survey method [7,8]. This method differs from traditional SP survey methods in that it can estimate an individual VOT, and the questions posed to participants are fewer and easier to understand. The overall outline of our experiment is as follows. It consists of two phases, both conducted on a computer. The first phase is based on the SABVSP survey method. After participants complete the survey, the individual VOT data derived from the survey is recorded. The second phase is traffic assignment over 3 links. It further consists of two parts. Part 1 implements our adaptive pricing policy, while part 2 implements marginal pricing based on the VOT estimated from part 1. In both the parts, the participants are instructed and incentivized to choose Nash flow with respect to the corresponding price as their traffic assignment. After both phases, participants are asked to fill out a questionnaire aimed at understanding their decision-making strategy.

The main contributions of the project are as follows. First, we provide an adaptive algorithm, with finite sample guarantee, to estimate VOT and link coefficient functions, in a routing game with polynomial link latency functions. Second, based on traffic assignment human subject experiments, we conclude that the VOT estimate using our method is lower than the one obtained by an existing adaptive stated preference method. Third, we show that the marginal pricing based on VOT estimated using our approach induces social equilibrium link flows in the experiments.

We conclude by summarizing key notations to be used throughout the report. For integers k_1, k_2, $k_1 \leq k_2$, $[k_1 : k_2]$ will succinctly denote the set of integers $\{k_1, k_1 + 1, \ldots, k_2\}$. When $k_1 = 1$, we shall use an even shorter notation of $[k_2]$ to denote $[1 : k_2]$. $\triangle_n^\delta(\lambda)$ shall denote the δ-interior of the n-simplex $\{x : x_i \in \lambda[\delta, 1-\delta],\ i \in [n]\ \sum_{i\in[n]} x_i = \lambda\}$. For brevity, the regular simplex $\triangle_n^0(\lambda)$ shall simply be denoted as $\triangle_n(\lambda)$. e_i denotes the i-th unit vector, where the total size will be clear from the context.

2 Problem Formulation

Consider a routing game over a parallel network with n links. Let the total demand by denoted by λ, and let the link latency functions be of the form:

$$\ell_i(x_i) = \sum_{j=1}^{m+1} a_{i,j-1} x_i^{j-1}, \quad i \in [n] \tag{1}$$

where x_i is the flow on link i. If $p = [p_1, \ldots, p_n]$ is the pricing on the links, and α is the value of time, then the total travel cost on the links is

$$u_i(x_i, p_i) = \alpha \ell_i(x_i) + p_i \tag{2}$$

For a given price p and demand λ, let $x^*(p, \lambda)$ denote the corresponding Nash flow.

The value of time and the coefficients of the link latency functions are unknown. We refer to them collectively as routing game parameters and denote as $\theta = \{\alpha, \{a_{i,j-1}\}_{i\in[n],\, j\in[m+1]}\}$. We wish to identify θ uniquely through adaptive pricing and parameter estimation algorithm. Let $\{p^{(k)} : k = 1, \ldots\}$ denote the prices in different trials, and let $\{x^{(k)} : k = 1, \ldots\}$ be the corresponding Nash flows, when the total demand during the corresponding trials are $\{\lambda^{(k)} : k = 1, \ldots\}$. That is

$$x^{(k)} = x^*(p^k, \lambda^{(k)}; \theta), \qquad k = 1, \ldots$$

where we explicitly emphasize the dependence of Nash flow on the unknown parameters. The rationale for varying λ over the trials will become clear when we present necessary conditions for identifiability. The adaptive pricing is allowed to be of the form $p^{(k)} \equiv p^{(k)}((p^{(s)}, x^{(s)}) : s \in [k-1])$, i.e., it depends on the past prices and the corresponding Nash flows. Let $\hat{\theta}_k \equiv \hat{\theta}_k((p^{(s)}, x^{(s)}) : s \in [k])$ be the estimates under the estimation algorithm. Our objective is to design an adaptive pricing policy and an estimation algorithm so that $\hat{\theta}_k \equiv \theta$ for all $k \geq N$, for some finite N.

It becomes clear from the necessary condition that we will be able to uniquely identify all parameters except free flow travel time on one of the links, say $a_{1,0}$. We do this by assuming knowledge of $a_{1,0}$ and estimating $\tilde{\theta} := \{\tilde{a}_{i,j-1} := \alpha a_{i,j-1}\}_{i\in[n],\, j\in[m+1]}$. The original parameters can then be estimated as:

$$\begin{aligned} \alpha &= \frac{\tilde{a}_{1,0}}{a_{1,0}} \\ a_{i,j-1} &= \frac{\tilde{a}_{i,j-1}}{\alpha} = \frac{a_{1,0}}{\tilde{a}_{1,0}} \tilde{a}_{i,j-1}, \qquad i \in [n],\ j \in [m+1] \end{aligned}$$

where the second equation is redundant for $a_{1,0}$. We recall from (1)-(2) that the total travel cost can be rewritten in terms of $\tilde{\theta}$:

$$u_i(x_i, p_i; \tilde{\theta}) = \sum_{j=1}^{m+1} \tilde{a}_{i,j-1} x_i^{j-1} + p_i, \quad i \in [n] \tag{3}$$

where we explicitly emphasize dependence on the unknown parameters to be estimated.

3 Parameter Estimation using Inverse Optimization

We adopt the following inverse optimization [2] for parameter estimation from k_0 most recent input-output samples $\{(p^{(s)}, x^{(s)}) : s \in [k-k_0+1:k]\}$. Specifically, $\hat{\hat{\theta}}_k$ is solution to:

$$\begin{aligned} \min_{\hat{\hat{\theta}}_k, y, \epsilon} \quad & \sum_{s \in [k-k_0+1:k]} \epsilon^{(s)} \\ \text{s.t.} \quad & u_i(x_i^{(s)}, p_i^{(s)}; \hat{\hat{\theta}}_k) \leq y^{(s)}, \qquad i \in [n],\, s \in [k-k_0+1:k] \\ & \sum_{i \in [n]} x_i^{(s)} u_i(x_i^{(s)}, p_i^{(s)}; \hat{\hat{\theta}}_k) \geq y^{(s)} - \epsilon^{(s)}, \qquad s \in [k-k_0+1:k] \end{aligned} \tag{4}$$

where $y = [y^{(k-k_0+1)}, \ldots, y^{(k)}]$, and $\epsilon = [\epsilon^{(k-k_0+1)}, \ldots, \epsilon^{(k)}]$. We treat k_0 as a free parameter, which will be specified later. It follows from (3) that (4) is a linear program.

4 Adaptive Pricing Algorithm

We start by providing a guideline for selecting k_0. It is easy to see that only $\epsilon \geq 0$ is feasible for (4). Therefore, if $\epsilon = 0$ is feasible, then it is also optimal. Indeed, $\epsilon = 0$ is feasible if the first set of inequalities in (4) all hold true with equality. This would be true if $x^{(k-k_0+1)}, \ldots, x^{(k)}$ all belong to the interior of $\triangle_n(\lambda^{(k-k_0+1)}), \ldots, \triangle_n(\lambda^{(k)})$ respectively. In this case, the second set of inequalities in (4) also hold true with equality. Therefore, plugging (3) into the constraints in (4) implies that $\hat{\hat{\theta}}_k$ is a solution to the system of linear equations: $X^{(s)}\hat{\hat{\theta}}_k = P^{(s)}$, $s \in [k-k_0+1:k]$, where:

$$X^{(s)} = \begin{bmatrix} e_1^T \otimes [1\, x_1^s\, \ldots\, (x_1^s)^m] \\ \vdots \\ e_n^T \otimes [1\, x_n^s\, \ldots\, (x_n^s)^m] \end{bmatrix}, \qquad \hat{\hat{\theta}}_k = \begin{bmatrix} \tilde{a}_{1,0}^{(k)} \\ \vdots \\ \tilde{a}_{1,m}^{(k)} \\ \vdots \\ \tilde{a}_{n,0}^{(k)} \\ \vdots \\ \tilde{a}_{n,m}^{(k)} \end{bmatrix}, \qquad P^{(s)} = \begin{bmatrix} y^{(s)} - p_1^{(s)} \\ \vdots \\ y^{(s)} - p_n^{(s)} \end{bmatrix}$$

Collecting all the samples, we get that

$$\underbrace{\begin{bmatrix} X^{(k-k_0+1)} \\ \vdots \\ X^{(k)} \end{bmatrix}}_{=:X_{k-k_0+1:k}} \hat{\hat{\theta}}_k = \underbrace{\begin{bmatrix} P^{(k-k_0+1)} \\ \vdots \\ P^{(k)} \end{bmatrix}}_{=:P_{k-k_0+1:k}} \tag{5}$$

(5) admits a unique solution in $\hat{\hat{\theta}}_k$ if $X_{k-k_0+1:k}$ is full column rank. The following gives a simple sufficient condition on $x^{(k-k_0+1)}, \ldots, x^{(k)}$ for this to happen.

Proposition 1. (5) *admits a unique solution* $\hat{\hat{\theta}}_k$ *if (i)* $k_0 \geq m+1$ *and (ii)* $x_i^{(k_1)} \neq x_i^{(k_2)}$ *for all* $k_1, k_2 \in [k-k_0+1, k]$, $k_1 \neq k_2$.

Proof. Without loss of generality, let $k_0 = m+1$. $X_{k-k_0+1:k}$ can be written as a block diagonal matrix $\mathbf{diag}(V_1, \ldots, V_n)$, where

$$V_i = \begin{bmatrix} 1 & x_1^{(k-m)} & \ldots & (x_1^{(k-m)})^m \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_1^{(k)} & \ldots & (x_1^{(k)})^m \end{bmatrix}, \qquad i \in [n]$$

are Vandermonde matrices. The result then follows from the fact that a Vandermonde matrix is non-singular if its arguments are all distinct from each other. □

We now design an adaptive policy which generates sufficient, as per Proposition 1, prices under which Nash flow is supported on all the links. We shall refer to such prices as *interior* prices. The policy consists of three phases. Phase 1 finds an interior price. Phase 2 generates a hypercube of interior prices centered at the price found in phase 1. Phase 3 samples the hypercube to generate sufficient interior prices for the estimation algorithm.

5 Human Subject Experiments

We performed a series of experiments with human subjects to investigate the efficacy of our approach from Section 4. We assumed the link coefficients to be known, and only the value of time to be unknown, for the experiments. The experiments also implement an existing approach for value of time estimation and compare its estimates against the estimates obtained using our approach. We describe, in Section 5.1, the experiment protocol as experienced by the participants. Section 5.2 describes the methodological foundation for the experiment design, and Section 5.3 presents findings from the experiments.

5.1 Experiment Protocol

The experiment protocol was reviewed and approved by the Institutional Review Board at the University of Southern California (USC # UP-24-00104). The experiment was conducted in the Kaprielian Hall at USC during March 2024 and April 2024. A total of 14 participants with a good command of the English language and with no prior experience with our experiment were recruited from the undergraduate and graduate population of the

Algorithm 1 Phase 1: find an interior price point

Input: $\delta_1 > 0$, $\epsilon_1 > 0$, $p_{\text{min}} \in \mathbb{R}^n$, $p_{\text{max}} \in \mathbb{R}^n$
Output: interior price point $p_{\text{int}} \in \mathbb{R}^n$

$p_{\text{int}} \leftarrow (p_{\text{min}} + p_{\text{max}})/2$
$\epsilon \leftarrow \epsilon_1$
while $x^*(p_{\text{int}}) \notin \triangle_n^{\delta_1}(\lambda)$ **do**
  for $i \in [n]$ **do**
    if $x_i \in \lambda[0, \delta_1)$ **then**
      $p_{\text{new},i} = p_{\text{int},i} - \epsilon$
    else if $x_i \in \lambda[1 - \delta_1, 1]$ **then**
      $p_{\text{new},i} = p_{\text{int},i} + \epsilon$
    end if
  end for
  $x_{\text{new}} \leftarrow x^*(p_{\text{new}})$
  $x \leftarrow x^*(p_{\text{int}})$
  if $\exists i \in [n]$ s.t. $x_i \in \lambda[0, (1 - \delta_1))$ and $x_{\text{new},i} \in \lambda[1 - \delta_1, 1]$ or $x_i \in \lambda(\delta_1, 1]$ and $x_{\text{new},i} \in \lambda[0, \delta_1]$ **then**
    $\epsilon \leftarrow \epsilon/2$
  else
    $p_{\text{int}} \leftarrow p_{\text{new}}$
  end if
end while

Algorithm 2 Phase 2: find a hyper-rectangle of interior price points

Input: $\delta_2 > 0$, $\epsilon_2 > 0$, interior price point $p_{\text{int}} \in \mathbb{R}^n$ from Phase 1
Output: centroid $c \in \mathbb{R}^n$ and size $\epsilon > 0$ of interior price hyper-cube, set of interior price points $\mathcal{P}_{\text{int}}$

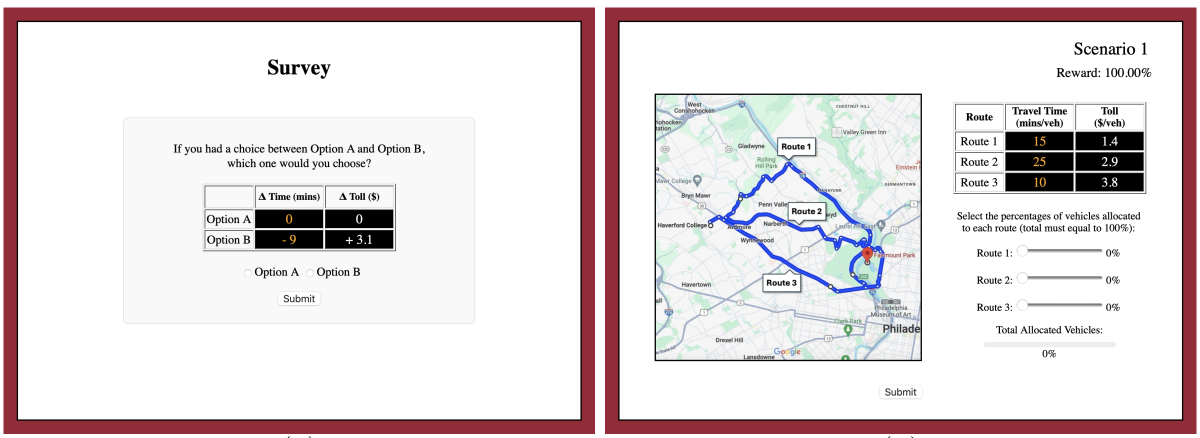
$\epsilon \leftarrow \epsilon_2$
$\mathcal{P}_{\text{int}} \leftarrow \{p_{\text{int}}\}$
$c \leftarrow p_{\text{int}}$
while $|\mathcal{P}_{\text{int}}| < m + 1$ or $\exists$ vertex of `hypercube`(c, ϵ) which is not interior price point **do**
  $\mathcal{P}_{\text{int}} \leftarrow \mathcal{P}_{\text{int}} \cup \{$vertices of `hypercube`$(c, \epsilon)\}$ which are interior price points
  $c \leftarrow$ `centroid`$(\mathcal{P}_{\text{int}})$
  $\epsilon \leftarrow \epsilon/2$
end while

Algorithm 3 Phase 3: generate sufficient interior price points

Input: $c \in \mathbb{R}^n$, $\epsilon > 0$, and $\mathcal{P}_{\text{int}}$; all from Phase 2
Output: $\mathcal{P}_{\text{int}}$ with $|\mathcal{P}_{\text{int}}| \geq m + 1$

while $|\mathcal{P}_{\text{int}}| < m + 1$ **do**
    $P_{\text{int}} \leftarrow P_{\text{int}} \cup \texttt{rand}(\texttt{hypercube}(c, \epsilon))$
end while

university. Upon arrival at the laboratory, participants were given an orientation presentation [9] explaining the protocol and objectives of the experiment. The experiments were conducted on a networked desktop computer. Except for experiment personnel, only one participant was present in the laboratory during an experiment session.



(a) (b)

Figure 1: Experiment user interface during (a) survey phase and (b) traffic assignment phase

The experiment consisted of two phases: a survey phase and a traffic assignment phase. In the survey phase, the participants are asked to answer a series of questions through a computer interface; see Figure 1(a). These questions are designed to get insight into the trade-offs between travel time and toll for the participants. In each question, the participant is asked to choose between options A and B, based on the time savings (△ time) vs. extra tolls (△ toll). For instance, for the question illustrated in Figure 1(a), choosing option B would result in time saving of 9 mins but at the expense of paying \$ 3.1 in toll when compared with option A. Without loss of generality, all the questions offer values for option B relative to option A. The questions are adaptive, i.e., values for option B in a question depend on the participant response in the previous question. Details are provided in Section 5.2.1. There is no time limit for answering any question. The survey phase ends when the the value of time is estimated with sufficient accuracy. On an average, this phase lasted for 8 questions.

The route selection phase consists of two parts. The user interface for both parts is shown in Figure 1(b). Each part consists of a series of scenarios in which the participant is asked to do traffic assignment based on the information provided. The information consists of a visual display of a traffic network consisting of three routes between a single origin destination pair. For each route, the interface provides information about travel time and tolls. Travel time is determined by affine link latency functions with coefficients given by:

$$a = \begin{bmatrix} 15 & 20 \\ 25 & 25 \\ 10 & 30 \end{bmatrix} \tag{6}$$

The participant does not know these coefficients, not do they know the total demand. The participant specifies traffic assignment through sliders, one for each route, in terms of percentage of total demand assigned to each route, and sees the corresponding change in travel time. In part 1, the tolls are set to be independent of the participant's traffic assignment. In part 2, the tolls are determined according to marginal pricing, and therefore change in response to the traffic assignment. The marginal pricing is given by:

$$p_i^{\text{marginal}} = \alpha a_{i,1} x_i, \qquad i = 1, 2, 3 \tag{7}$$

where α is the estimate obtained from analysis of the traffic assignments in part 1; see Section 5.2.2 for details. The participant can try different assignments and see the corresponding changes in travel times (and also tolls in phase 2) as many times as they wish before finalizing one by pressing the "Submit" button. There is no time constraint for finalizing the traffic assignment, and the only requirement is that the sum of % chosen for each of the routes is equal to 100 %.

For both the parts in the assignment phase, the participants are instructed to assume the role of an operator of a fleet of (freight) vehicles who is dispatching the vehicles among different routes. The drivers of the vehicles are individually responsible for the transportation time as well for the toll they pay for their trip. The participant is instructed to be fair when assigning routes to the fleet vehicles so as to be *fair* to the drivers by balancing their travel times and tolls according to its own value of time. It is expected that this framing of roleplaying will translate into the participant's traffic assignment corresponding to user equilibrium. In order to reinforce this behavior, the participants are displayed a reward % on the user interface which is updated after every scenario. This reward % is reflective of the consistency between the value of time estimates from the survey phase and from the previous scenarios in the traffic assignment phase. Details are provided in Section 5.2.2. Phase 1 of the traffic assignment phase lasts for 40 scenarios and phase 2 lasts for 20 scenarios.

A participant is not allowed to skip any question in the survey phase nor are they allowed to skip any question in the traffic assignment phase. After both the phases, the participant is asked to fill a paper based survey to get insight into their traffic assignment strategies during their experiment. This survey is provided in the Appendix. Every participant received a show-up reward of $10 and an additional maximum of $10 depending on the reward % at the end of all the scenarios. All renumeration was paid in cash. The average total renumeration per participant came out to be $17. A participant session lasted for approximately 90 minutes on average.

5.2 Methodological Background

In this section, we describe the methodological foundations for the two phases of the experiment.

5.2.1 Survey Phase

Recall that the questions in the survey phase are adaptive to the participant's responses to previous questions. This is based on the SABVSP method, which is a survey method specifically designed to assess the VOT of drivers associated with using toll roads. The survey consists of a series of stated preference questions that present a choice between two routes: one is a free route, and the other is a toll route that, while incurring a specific charge, offers a certain travel time saving. SABVSP does not require respondents to evaluate all possible combinations of tolls and travel time savings. Instead, it uses a pre-determined matrix in conjunction with the respondent's initial answers to infer other potential responses, thus reducing the overall number of questions and optimizing the sequence of questions.

Table 1: Options Matrix

Toll	Time Savings (Minutes)				
	3	6	9	12	15
$5.50	-	-	-	-	-
$4.90	-	-	-	-	-
$4.30	-	-	-	-	-
$3.70	-	-	-	-	-
$3.10	-	-	-	-	-
$2.50	-	-	-	-	-
$1.90	-	-	-	-	-
$1.30	-	-	-	-	-
$0.70	-	-	-	-	-
$0.10	-	-	-	-	-

Table 2: Initial Answers Matrix after Question 1

Toll	Time Savings (Minutes)				
	3	6	9	12	15
$5.50	-	-	-	-	-
$4.90	-	-	-	-	-
$4.30	-	-	-	-	-
$3.70	-	-	-	-	-
$3.10	-	-	A	-	-
$2.50	-	-	-	-	-
$1.90	-	-	-	-	-
$1.30	-	-	-	-	-
$0.70	-	-	-	-	-
$0.10	-	-	-	-	-

SABVSP is straightforwardly implemented in our experiment, with the interpretation that option A corresponds to free route and option B corresponds to tolled route. The method uses the pre-determined options matrix shown in Table 1. The first question for every participant is the one from the middle of the options matrix, i.e., \$3.10 additional toll and 9 minutes of time savings for option B. The participant's response to this question is assumed to imply response to a subset of other questions. For example, if the response is option A as shown in Table 2, then this implies the response will also be option A to other questions illustrated in Table 3. The assumption is that the participant would be equally willing to pay the same toll for greater time savings or would be willing to pay lower tolls for the same time savings. The next question is selected by arranging the remaining options starting with bottom left, then from left to right and bottom to top, and picking the middle entry of this array. For example, in Table 3, this corresponds to \$1.90 toll and 9 minutes of time savings for option B. The process is repeated until the entire matrix is filled up, e.g., as shown in Table 4.

Table 3: Final Answers Matrix after Question 1

Toll	Time Savings (Minutes)				
	3	6	9	12	15
\$5.50	A	A	A	-	-
\$4.90	A	A	A	-	-
\$4.30	A	A	A	-	-
\$3.70	A	A	A	-	-
\$3.10	A	A	A	-	-
\$2.50	-	-	-	-	-
\$1.90	-	-	-	-	-
\$1.30	-	-	-	-	-
\$0.70	-	-	-	-	-
\$0.10	-	-	-	-	-

Table 4: Final Answers Matrix after All Questions

Toll	Time Savings (Minutes)				
	3	6	9	12	15
\$5.50	A	A	A	A	A
\$4.90	A	A	A	A	A
\$4.30	A	A	A	A	A
\$3.70	A	A	A	A	A
\$3.10	A	A	A	A	A
\$2.50	A	A	A	B	B
\$1.90	A	A	B	B	B
\$1.30	A	B	B	B	B
\$0.70	A	B	B	B	B
\$0.10	B	B	B	B	B

Next, the method identifies turning points in the answer matrix to determine the indifference values at which participant switches from being unwilling to being willing to pay a lower toll for the same time saving. The *indifference* value for that time saving is then taken to be the mean of the unwilling and willing toll values. For example, referring to Table 4, the indifference value for time savings of 9 minutes is $(\$2.5 + \$1.9)/2 = \$2.2$. These indifference values are then plotted, and each participant's VOT is equal to the slope of the best-fit line, e.g., see Figure 2. This estimate of VOT will be denoted as $\hat{\alpha}^{\text{survey}}$.

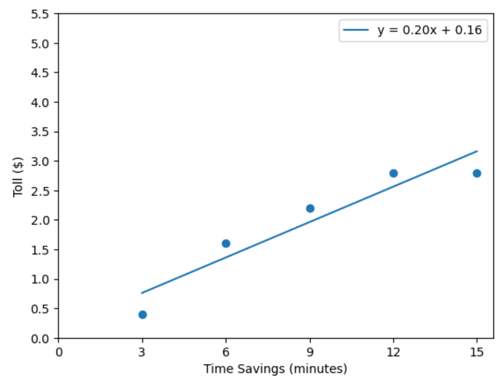


Figure 2: Indifference values of tolls for different time savings for a sample participant. The slope of the best-fit line, i.e., 0.2 $/min, is taken to be survey-based VOT estimate for this participant.

5.2.2 Traffic Assignment Phase

Recall that the total number of scenarios in parts 1 and 2 is equal to 60. The values of total demand λ are repeated for two consecutive scenarios. Therefore, $\lambda^{(2k)}$, $k \in [30]$, are i.i.d sampled from uniform distribution with support $[1, 5]$. We also repeat the link-wise price values for two consecutive scenarios in part 1. Specifically, the prices $p^{(2k)}$, $k \in [20]$, are i.i.d sampled from uniform distribution with support $[1, 5]^3$. Note that these values correspond to the last phase of our adaptive pricing algorithm in Section 4. Recall that the prices in part 2, i.e., $p^{(2k)}$, $k \in [21 : 30]$, are based on marginal pricing. However, since the corresponding demands are repeated over two consecutive scenarios, it is natural to expect repeatability in the ensuing traffic assignment chosen by the participant.

Reward Calculation: Recall that the reward % is the information provided to the participant to reinforce their traffic assignment decision to resemble user equilibrium in a way that is consistent with the value of time that they exhibit during the survey phase. Let $x^{(k)} \in \mathcal{S}_3(\lambda^{(k)})$ be the traffic flow induced by the assignment choice made by the participant in scenario $k \in [60]$. Note that this is equal to the fractions chosen by the participant multiplied by the total demand $\lambda^{(k)}$ which is unknown to the participant. Let $x^*(p^{(k)}, \lambda^{(k)}; \hat{\alpha}^{\text{survey}}) \in \mathcal{S}_3(\lambda^{(k)})$ be the user equilibrium in scenario k that is consistent with the value of time exhibited during the survey phase. Therefore, a natural choice for *penalty* in scenario k is proportional to $\|x^{(k)} - x^*(p^{(k)}, \lambda^{(k)}; \hat{\alpha}^{\text{survey}})\|_2$. The reward % to be displayed in scenario k is therefore set to be equal to $100 - 50\frac{\sum_{i=1}^{k-1} \|x^{(i)} - x^*(p^{(i)}, \lambda^{(i)}; \hat{\alpha}^{\text{survey}})\|_2}{k-1}$, with the convention that the reward % for scenario 1 is equal to 100. The proportionality constant of 50 was chosen to ensure that the reward % was sufficiently sensitive to provide quick feedback to the participant when they deviated from the user equilibrium behavior. Note that the reward % is cumulative over all past decisions, and hence may become less sensitive to the deviations in later scenarios. Moreover, recall that part of the renumeration is determined by final reward % which is cumulative over all scenarios. This does not distinguish between participants exhibiting consistent deviations versus those whose deviation profile is erratic. Alternate choices of computing reward, e.g., using discounted average, will be explored in future work.

5.3 Experiment Findings

Our objectives for the experiment were the following:

(O1) Investigate consistency between the value of time estimated using the traditional survey based method and our online adaptive algorithm in Section 4; and

(O2) Investigate consistency between the participant decision under marginal best pricing (based on estimated value of time) and social equilibrium

We recorded the following data during the experiment for each participant, for each question in the survey phase and for each scenario in the traffic assignment phase: (i) information displayed on the screen; (ii) final responses of the participant, i.e., when they press "Submit"; (iii) start and end times of each question/scenario. In addition, we recorded response to paper-based survey questions from a subset of the participants. We now describe our findings from the analysis of these data towards the two main objectives of the experiment.

5.3.1 Comparing value of time estimates

Figure 3 compares the value of time estimates using the SABVSP method (survey phase) and using our adaptive approach (traffic assignment phase), for some representative participants. The estimate from the survey phase is shown as a constant value, whereas the estimate from the assignment phase is expectedly time-varying. Note that the latter estimate starts from scenario $k = 3$. Recall that, for the experiments, we assume that the link coefficients are known and only VOT is unknown. One can then show similar to Proposition 1 that only one sample is sufficient for VOT estimation. Considering repetition in demand and price values for two successive samples, we therefore estimate VOT starting with $k = 3$. Subsequently, after the k-th scenario, $k \geq 3$, we use samples from all past k scenarios for VOT estimation.

The plots indicate that the value of time estimate using our adaptive approach is consistently lower than the one obtained from the survey approach. The gap between the two estimates varies across the participants. Note the transient in the value of time estimates during the first few scenarios – this could be attributed to the the participants adjusting to the switch from the survey phase to the assignment phase. The estimates to approximately

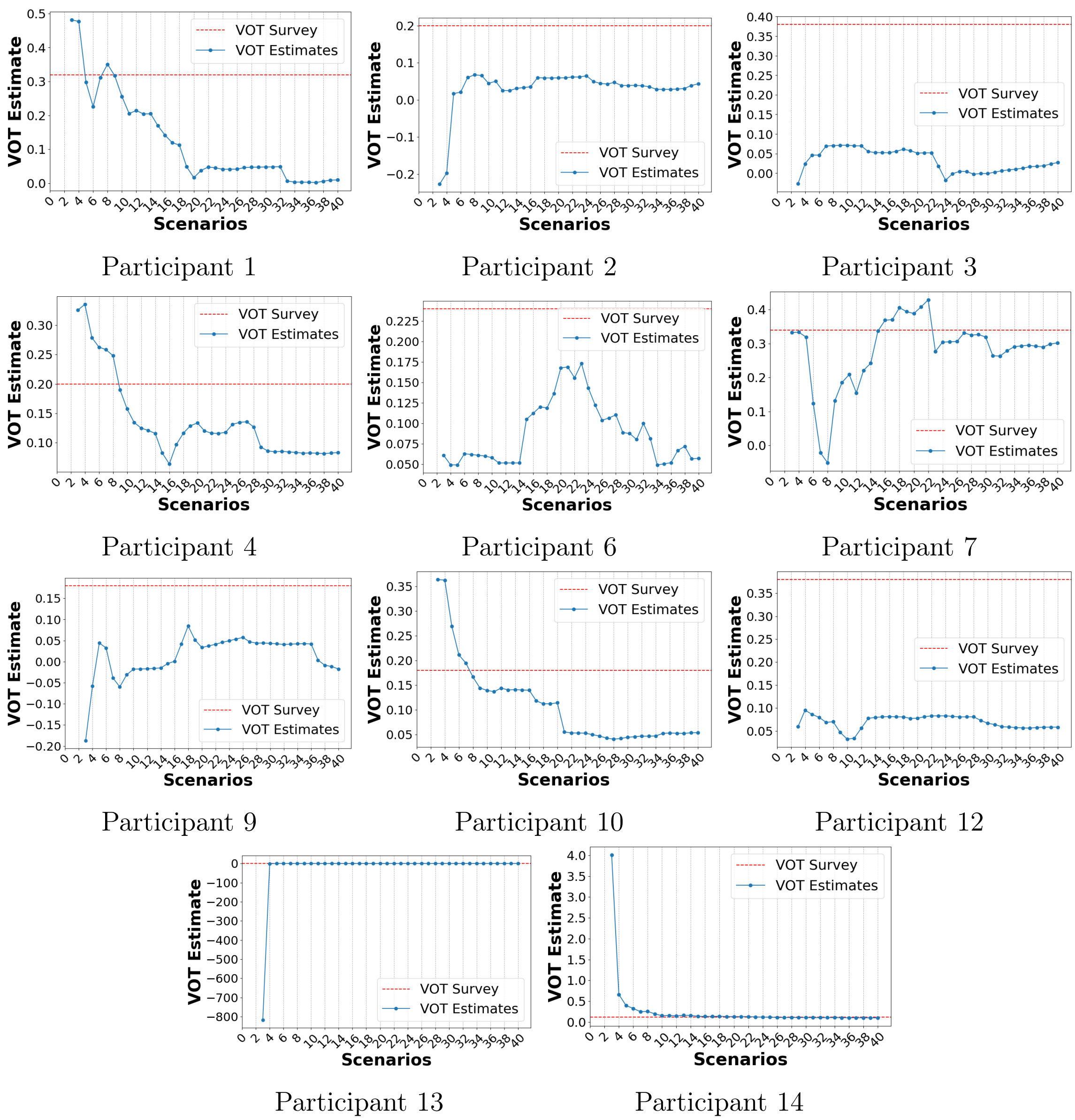


Figure 3: Comparison of value of time estimates, in $/minute, for representative participants, for the survey based vs. our adaptive approach.

steady state value towards the end of part 1 of the assignment phase, i.e., by scenario 40. The distributions of these "steady-state" values across participants are compared against the distribution of the estimates from the survey approach in Figure 4. As before, these distribution plots suggest the value of time estimate has a higher mean, but also a higher variance in comparison to the estimates using our adaptive approach. Note that the the

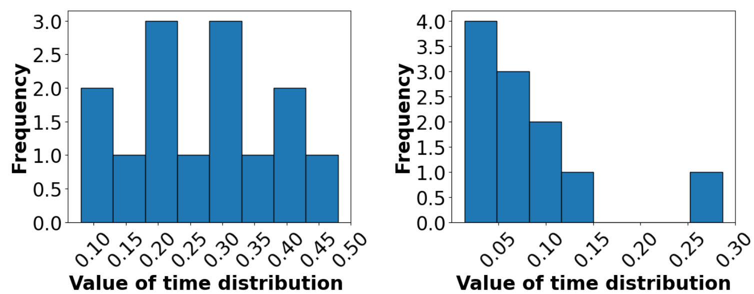


Figure 4: Distribution of value of time estimates, in $/min, across all participants obtained from (a) survey approach, and (b) our adaptive approach.

VOT estimates show transient effect in the earlier scenarios for some participants, possibly due to them adjusting to the switch from the survey phase to the traffic assignment phase. The VOT estimates for 3 out of 14 participants were consistently negative, possibly because of their improper understanding of their objectives in the experiment. This was consistent with very low reward % for these participants. These participants did not proceed to the second part (traffic assignment under marginal pricing) and their data is removed from our reporting.

5.3.2 Does marginal pricing result in social equilibrium?

In part 2 of the assignment phase, we use marginal pricing in (7) with α corresponding to the average of the VOT estimates from $k = 31$ to $k = 40$ in part 1. Recall that $x^{(k)} \in \mathcal{S}_3(\lambda^{(k)})$ denotes the link flows induced by the assignment choice made by the participant in scenario k. Let $x^{\text{soc}}(\lambda)$ denote the link flows under social equilibrium when the total demand is λ. Figure 5 shows the percentage deviation between the *expected* link flows based on the value of time estimates from part 1 with respect to actual link flows, i.e., $\frac{\|x^{(k)} - x^{\text{soc}}(\lambda^{(k)})\|_2}{\|x^{\text{soc}}(\lambda^{(k)})\|_2} \times 100$, for the scenarios in part 2.

The consistently low percentage deviation in Figure 5 indicates the effectiveness of marginal pricing in inducing social equilibrium in link flows, and indirectly also suggest that the participants exhibit consistency between their values of time when the congestion pricing is fixed vs when it is also adaptive (according to marginal pricing rule).

6 Conclusion and Future Work

In this project, we investigated theoretical and empirical basis for estimating parameters of a routing game, with particular emphasis on the value of time, using an adaptive algorithm. Our estimation algorithm has finite sample guarantee in the ideal case, and provides converging estimates for VOT in human subject traffic assignment experiments. Interestingly, these estimates are consistently found to be lower than the one obtained from an existing adaptive stated preference method aimed at estimating VOT for an individual participant. This could be because of the increase in the complexity of the traffic assignment decision setup versus the relatively simple dual choice decision setup in the stated preference method. On the other hand, the marginal pricing using the VOT estimates using our approach was found to be quite effective in inducing social equilibrium link flows. This implies that an effective real-time pricing should use VOT estimated using an (adaptive) algorithm such as ours using a similar decision setup, as opposed to estimates from survey based methods. In future, we plan to extend our adaptive pricing algorithms and experiments to time of departure [10] and other important transportation decisions, and eventually to integrate with the information design [11–14].

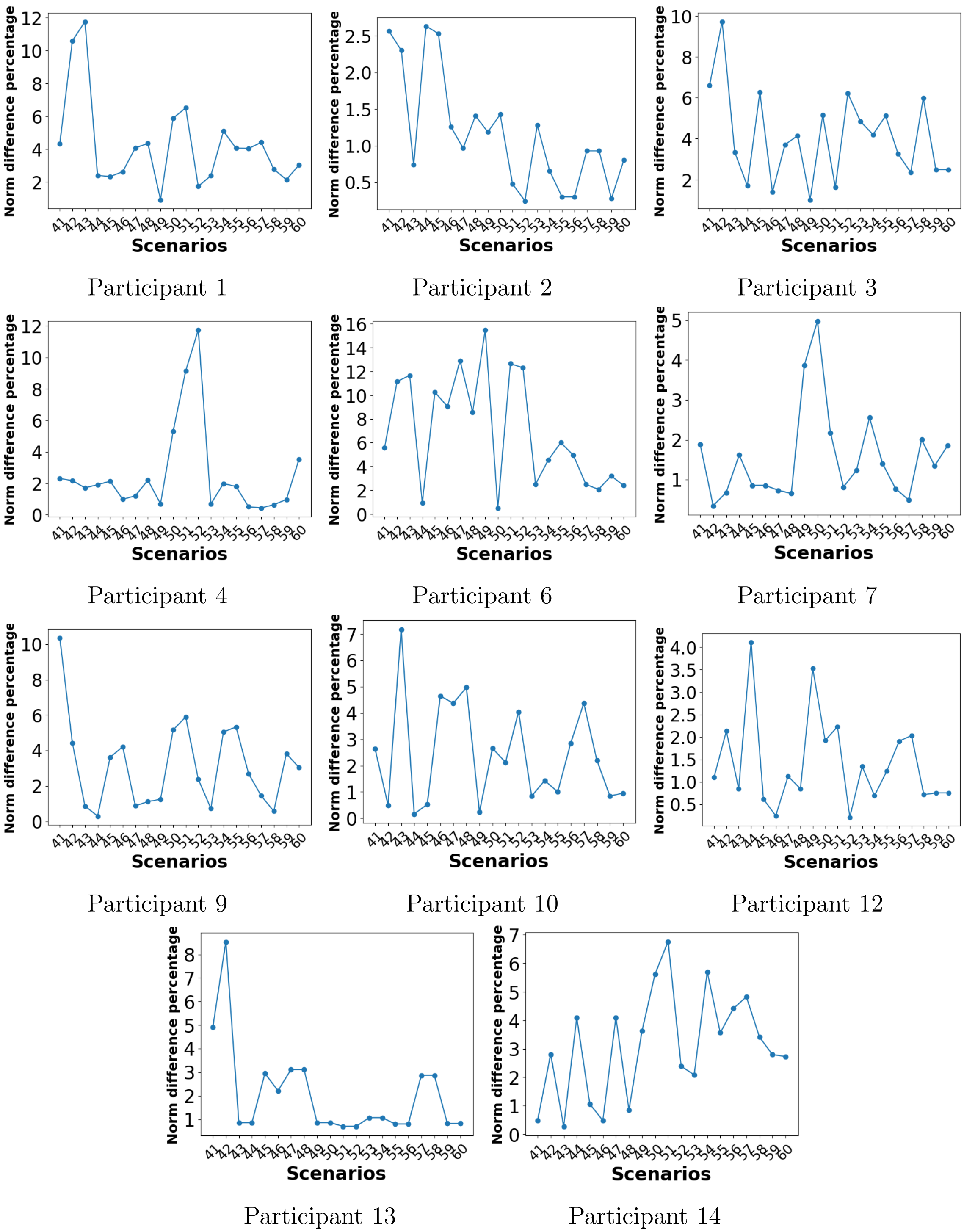


Figure 5: Percentage deviation between social equilibrium and the observed link flows in part 2 of the assignment phase, for representative participants.

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7 Data Management Plan

Products of Research

Human subject experiment data cannot be shared as per the IRB protocol. The analysis from that data is included in the report. Additionally, simulation data was generated for the adaptive pricing algorithm.

Data Format and Consent

All the simulations were done in Python.

Data Access and Sharing

The input/output data for the simulations in this report is available at `https://viterbi-web.usc.edu/~ksavla/code.html`.

Reuse and Redistribution

The data can be reused freely for non-commercial purposes. Its usage, in original or after modification, in publications is to be done with due acknowledgement to the authors of this report and by citation of relevant publications by the authors.

Appendix: Questionnaire

1. On the scale of 1 to 5 (with 5 being the highest), how did you feel you understood the questions in the survey part?
2. On the scale of 1 to 5 (with 5 being the highest), how did you feel you understood what was happening in each scenario of Phase 1 (the first 40 scenarios), what you were doing and what you needed to do next?
3. On the scale of 1 to 5 (with 5 being the highest), how did you feel you understood what was happening in each scenario of Phase 2 (the last 20 scenarios), what you were doing and what you needed to do next?
4. On the scale of 1 to 5 (with 5 being the highest), how often did you check the reward percentage before selecting fractions?
5. When you indeed checked the reward percentage, on the scale of 1 to 5 (with 5 being the highest), how much did the reward percentage affect your fraction selections?
6. How did you make your fraction selections?
   (a) Based on careful consideration of trade-off between time and cost.
   (b) Paying attention mostly to the cost
   (c) Paying attention mostly to the travel time
   (d) Arbitrarily
7. If you chose A in question 6, what is your trade-off between time and cost?
8. If none of the four options in question 6 applies to you, then please describe your strategy for making fraction selection here.
9. What did you like (if any) or dislike (if any) about the experiment interface?
10. Any other comments you might have